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## C-MATRIX: A TRANSFORMATION OF SIGNALS

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### Abstract

In this work we present a matrix, C-Matrix, and its characteristics, as the output of C-Transform that we have just introduced and used in our previous works. Furthermore we'll illustrate some applications of our matrix in the field of P.R. by showing how much some features that we consider are strictly related to the signal to which we applied our transform. In a second phase we'll describe a matching between textures from which we had extracted some features by C-Matrix method and by methods presented by other authors.

### Introduction

Our work lies in the field of texture analysis, that is, as known, one of the most important aspects of P.R. (1,...,5). Infact, the problem is to describe texture in terms as rigorous possible, because it plays an important role in describing objects that belong our world. Actually they have not provided a full description of texture but in empirical and often approximate terms. Let us remember, to simplify, a definition that seem to be more accepted than others proposed for the attempt of describing texture: "an effect of texture is produced when a pattern consist of a high number of subpatterns with a size much smaller than that of the entire pattern. These subpatterns must exhibit a certain density and regularity, which might be coarse over the entire visual field. They may have also their own substructures". According to our method, we consider texture to be a signal (in this case two-dimensional because it is conceived by computer as an array whose size is  $M \times N$ ), then we apply a Transform (C-Transform)(6,...,10)

obtaining as output the C-Matrix, from which we may extract some features which we retain to be useful to the attempt of realizing all that we earlier said. Therefore our method belongs to the research using signal transformations for displaying its characteristics (11,12).

### C-Matrix

In this section we introduce some basic concepts for interpreting C-Matrix. On the set representing the range of the signal, we make a run of partitions into subsets whose dimension vary between 1 and  $m$ , where  $m$  is the width of the range, computing step by step the differences between maximum and minimum grey values that signal exhibits in each element of the partition itself.

Let us now remember that we refer digitized signals and that, for shortness, we'll omit this adjective in the following. If we carry out, in a table, the frequency of the various differences between the extrema with respect to the sizes of the sets partitioning the range, we obtain a matrix, called C-Matrix, that really is the transform of the input signal. For a better understanding, let us note that rows indicate the successive sizes (according to an increasing order of the set "window") and columns indicate the dynamic of the signal, i.e. the differences introduced above.

In earlier works, we have illustrated the theoretical foundations of this method (6,7,8,9,13).

Here we'll briefly summarize the main differences we have introduced and advantages we have obtained with respect to the previous applications.

- 1) The scansion of two-dimensional signals goes area by area instead of row by row.
- 2) We have a different description of our experimental results which allows an

easier and better interpretation of them.  
From (1) we see that:

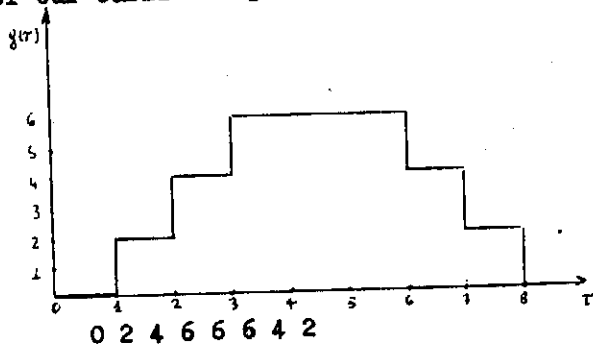
- a) the transformed signal is invariant to image rotation
- b) we can compute the spatial frequency
- c) we have a reduction of computation time.

- While from (2) we can
- i) eliminate the phase of representation of the data obtained in a 3-dimensional space, "C-Space",
  - ii) extract from the signal the smallest monotonical zone, the front of increment, of decrement, the maximal amplitude, even tual "plateaux", etc.

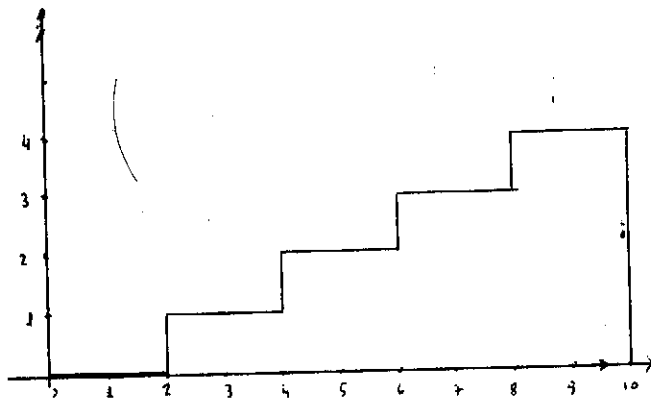
Because the operating way of our transformation is independent on the kind of signal to which we apply it, we'll consider, in our exemples, one-dimensional signals (i.e. those ones that are computer represented by an array of digits whose size is  $1 \times n$ ).

By observing a large number of artificial signals that we have just conceived and their own C-Matrices, we always noted a correspondance between the structural characteristics of the matrix (here we just mean the spatial placements of non zero elements through the matrix itself) and the signal characteristics from which we derived this matrix.

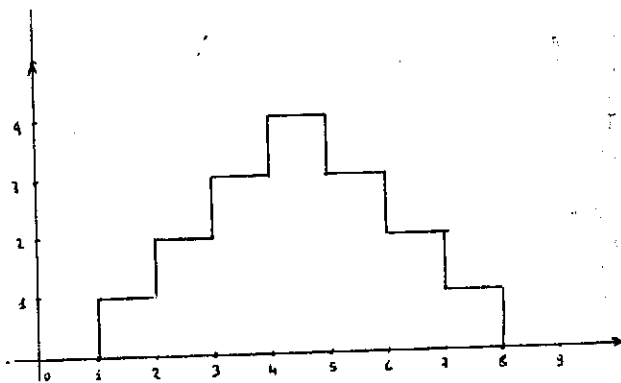
The following figures clarify the meaning of our earlier explanations.



	0	1	2	3	4	5	6
2	20	0	59	0	0	0	0
3	10	0	29	0	39	0	0
4	0	0	20	0	38	0	19
5	0	0	19	0	29	0	37
6	0	0	0	0	19	0	55
7	0	0	0	0	0	0	64
8	0	0	0	0	0	0	73
9	0	0	0	0	0	0	72
0	0	0	0	0	0	0	71



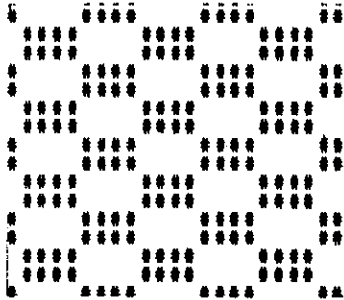
	0	1	2	3	4
2	50	40	0	0	9
3	0	80	0	0	18
4	0	40	30	0	27
5	0	0	60	0	36
6	0	0	30	20	45
7	0	0	0	40	54
8	0	0	0	20	73
9	0	0	0	0	92
10	0	0	0	0	91
11	0	0	0	0	90
12	0	0	0	0	89



	0	1	2	3	4
2	0	79	0	0	0
3	0	19	59	0	0
4	0	0	38	39	0
5	0	0	19	38	19
6	0	0	0	38	37
7	0	0	0	19	55
8	0	0	0	0	73
9	0	0	0	0	72
10	0	0	0	0	71

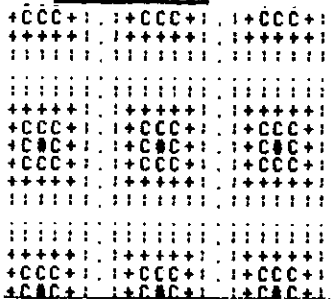
The sequence of numbers appearing in shaded form through the C-Matrix has been considered as a set of features and their formal definitions are shown in the table displayed in the following pages.

Let us note that the matrix hosts these features in different combinations depending exclusively on the type of the signal. At a first level the presence or the absence of these numerical strings identifies the kinds of fronts constituting the impulse, while at a deeper level, i.e. testing some characteristic parameters we are able to state the front behaviour. In order to verifying the general availability of the hypothesis we have formulated for unidimensional signal, we have realized a computer program that, by repeating unidimensional signal along certain directions, generates artificial textures. The C-Matrix we have computed for these textures, as we also can see from the figures carried out, preserves all the features which we just know.



Texture generated by signal 0 1

	0	1
2 x 2	384	577
3 x 3	0	900
4 x 4	0	841
5 x 5	0	784
6 x 6	0	729



Texture generated by signal 01234321

	0	1	2	3	4
2 x 2	0	861	0	0	0
3 x 3	0	187	413	0	0
4 x 4	0	0	376	463	0
5 x 5	0	0	175	392	0
6 x 6	0	0	0	352	377
7 x 7	0	0	0	163	513
8 x 8	0	0	0	0	625

Tables of features extracted from C-Matrix.

Notations.

In the following paper we indicate with:

- $[C]_{m,n}$  the C-Matrix
- $r_h$  the h-th row of  $[C]_{m,n}$   $1 \leq h \leq m$ ,
- $c_k$  the k-th column of  $[C]_{m,n}$   $1 \leq k \leq n$ ,
- $a_{h,k}$  the element of  $[C]_{m,n}$  which crosses between the row h-th and the column k-th,
- F.1) the feature i-th.

Let us display now some of the main features that may be extracted from an indefinite matrix.

F.1)

$$F.1a) = 0 \quad \text{if} \quad \nexists S_1$$

$$F.1a) = 1 \quad \text{if} \quad \exists S_1$$

where:

$$S_1 = \{s_i = a_{h,k} \in [C]_{m,n} / k = \max_{i=1, \dots, n} a_{h,k} \neq 0\}$$

for  $h=1, \dots, m \wedge \forall s_i: s_i = a_{1,j}, s_{i+1} = a_{1+\xi, j+\eta}$   
 $\wedge s_i - s_{i+1} = \tau$  with  $\xi, \eta, \tau$  constants  $\neq 0$

F.1b) = 0 if  $\nexists S_2$   
 F.1b) = 1 if  $\exists S_2$

where:

$$S_2 = \{s_i = a_{h,k} \in [C]_{m,n} / k = \max_{i=1, \dots, n} a_{h,i} \neq 0\}$$

for  $h = 1, \dots, m \wedge$

$\wedge \forall s_i: s_i = a_{1,j}, s_{i+1} = a_{1+\theta, j+\delta}$   
 with  $\theta, \delta$  constants  $\neq 0$ .

$$\begin{aligned} F.1c) &= 0 & \text{if } \nexists S_3 \\ F.1c) &= 1 & \text{if } \exists S_3 \end{aligned}$$

where:

$$S_3 = \left\{ s_1 = a_{1,k}, \dots; s_t = a_{t,k} / k=n, a_1 \neq 0 \wedge \wedge \exists \exists / a_{t,k} \neq 0 \in r_t \right\}$$

$$\begin{aligned} F.1d) &= 0 & \text{if } \nexists S_4 \\ F.1d) &= 1 & \text{if } \exists S_4 \end{aligned}$$

where  $S_4$  is the set extracted from  $[C]_{m,n}$  whose elements have some properties given by various combinations of those that characterize the elements of the sets  $S_1$  and  $S_2$ .

$$\begin{aligned} F.2) \\ F.2a) &= 0 & \text{if } \nexists D_1 \\ F.2a) &= 1 & \text{if } \exists D_1 \end{aligned}$$

where:

$$\begin{aligned} D_1 = \{ d_i = a_{r,s} \in [C]_{m,n} / \forall (s_1, s_{i+1}) \in \\ \in S_1 - \{s_n\} : s_i = a_{q,s}, s_{i+1} = a_{q+\xi, s+\eta} \\ \Rightarrow \exists (d_i, d_{i+1}) \exists d_i = a_{q',s} \text{ and} \end{aligned}$$

$$d_{i+1} = a_{q'+2\xi, s+\eta}$$

whith  $\xi, \eta$  constants  $> 0 \wedge$

$$\wedge (d_i - d_{i+1}) = 0 \wedge a_{r-1,s} \neq d_i \neq a_{r+1,s}$$

$$F.2b) = 0 \quad \text{if } \nexists D_j \text{ for } j=2, \dots, k \text{ which } k < p \quad (.)$$

$$F.2b) = i = \max j \quad \text{if } \exists D_j \text{ for } j=2, \dots, k \text{ which } k < p \quad (.)$$

where:

$$D_j = \left\{ d_i^j = a_{r,s} \in [C]_{m,n} / \text{if } \forall (d_i, d_{i+1}) \in D_1 : \right.$$

$$d_i = a_{q,s} \text{ and } d_{i+1} = a_{q+\xi, s+\eta} \Rightarrow$$

$$\Rightarrow \exists (d_i^j, d_{i+1}^j) \exists d_i^j = a_{q',s} \text{ and}$$

$$d_{i+1}^j = a_{q'+\xi, s+\eta}$$

whith  $\xi, \eta$  constants  $> 0,$

$$\wedge (d_i^j - d_{i+1}^j) = 0 \wedge a_{r-1,s} \neq d_i^j \neq a_{r+1,s}$$

(.)(.) p and  $\delta$  are two parameters depending on the presence or on the lacking

of the following feature; they may be obtained by knowing the set marking it.

$$\begin{aligned} F.3) \\ F.3) &= 0 & \text{if } \nexists P \\ F.3) &= 1 & \text{if } \exists P \end{aligned}$$

where P is defined as follows:

$$P = \left\{ p_i = a_{h,1} / a_{h,1} > 0, h=1, \dots, m \right\}$$

The results we have obtained are leading us, therefore, to proceed along this way; to this purpose we have considered a certain number of natural textures (in a digitized form) and we are obtaining now some experimental results that we'll interpretate in a next time.

An other application of C-Matrix.

At the some time we have made another work, by giving a different interpretation of the elements in C-matrix. Beginning from a number of natural texture images, we have applied three different algorithms to feature extraction from them for a successive automatic classification.

The first method (14) consists in considering as texture feature the squared euclidean distance:

$$\begin{aligned} H^2(I', I'') &= \sum_{i=p}^{M-n} \sum_{j=q}^{N-n} (a_{ij} - a_{i+r, j+r})^2 = \\ &= E(\tau, \theta) \end{aligned}$$

and constructing an array  $X_R$  whose elements are these distances depending from two different parameters.

The second method (15), the one called "co-occurrence matrix method", uses a set of 4 measures considered as characteristics of the examined texture. From these ones you may extract a number of features, as contrast, entropy, maximum cooccurrence frequency, etc., related to a particular texture.

In this second case we realized a feature array, too.

The third method (16,17,18,19), that we call "C-Matrix method" works, as the

preceding ones, on a digitized image

$$I = \|a_{i,j}\| \quad i=1, \dots, m; \quad j=1, \dots, n$$

with  $L$  grey levels. On this image we operate transformation, C-Transform, along the some lines that we have precedently described, to get a C-Matrix.

From this matrix we note that if texture is busy, correspondently to smale scansion submatrices i.e. through the beginning rows we have maximum frequencies in high values of the dynamic of the signal.

Indeed, when texture exhibits uniform regions quite large, C-Matrix elements, whith respect to small sizes of "window", are very high.

From these considerations, we have considered as features those ones that emphasize element distribution in the C-Matrix. From C-Matrix we have totally extracted 27 features, syntetizing them in the following three classes:

a) feature referring to the successive distances of first non zero elements appearing in each row of the matrix, with respect to 0 column

b) features refering to an ordered succession, row by row, of maximum values of frequency in the C-Matrix,

c) features refering to window width to which corresponds maximum absolute values of frequency.

The typical property of features in our method is essentially that they are not statistics, in opposition to the ones extracted by the other two methods we have said about; therefore our features are non sensitive to random phenomena (noise).

The results we obtained by applying a clustering algorithm to feature array, an output of each method experimented, allowed us to verify the goodness of our ipotesis in most of the cases we have analysed.

Statistical comparings and conclusions.

To realize a comparing between the methods, we have translated into correlation tables the experimental results obtained and computed Pearson-Bravais index that has provided us the correlation coefficient between the three methods, every time considered two by two.

We have obtained the following results:

$$r_{I,II} = 0,88$$

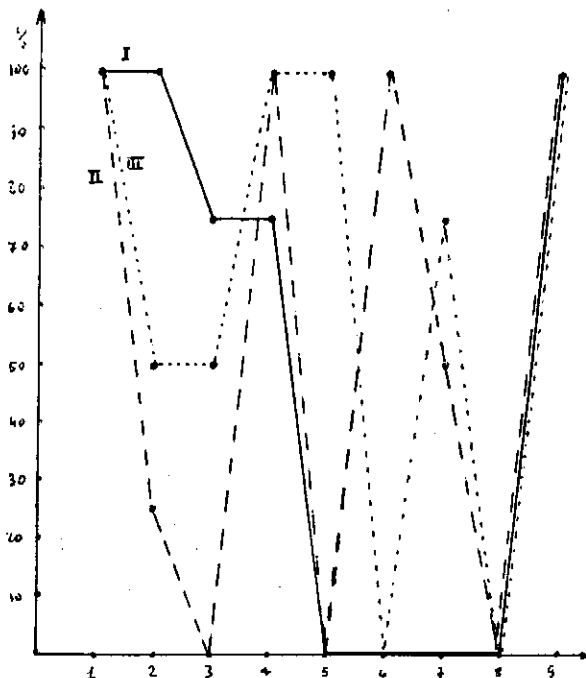
$$r_{II,III} = 0,93$$

$$r_{I,III} = 0,89$$

As we can note from the table and graphic below the method of co-occurrence matrices and that of C-Matrix are much more correlated each other than to the method of shifted submatrices.

From the graphic we also can note what are the cases in which each method has failed down.

	I	II	III
1	100%	100%	100%
2	100%	25%	50%
3	75%	0%	50%
4	75%	100%	100%
5	0%	0%	100%
6	0%	100%	0%
7	0%	50%	75%
8	0%	0%	0%
9	100%	100%	100%



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