Fuzzy modeling and evaluation of an ITM user

Luigi Di Lascio - Enrico Fischetti - Antonio Gisolfi

Dipartimento di Matematica e Informatica
Università di Salerno
E-mail: gisolfi@dia.unisa.it

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itable model of an ITM user is presented in s of concepts and mastery of abilities. Then it own how, via a fuzzy algebra, it is possible to itor and evaluate user's cognitive and nological states.

words

y sets, fuzzy variable, linguistic variable, / algebraic structure, evaluation, linguistic oximation.

troduction

nning in about 1960, Artificial Intelligence attempted to build computerized systems that l perform in the same way as human beings. ne 1970s researchers attempted to develop igent computer-assisted instruction systems, said ITSs (acronym for Intelligent Tutoring ems). ITSs take into consideration the idual differences of each user and provide mic individualization of instruction. ever, even with the unquestionable success of in certain environments [Sleeman and vn, 1982, Katz *et al.*, 1992, Ohlsson, 1987], verall performance of these systems were not uate [Self, 1990]. ITMs (Intelligent Tutoring imedia) are the evolutionary successors of and they heavily rely on the recent dramatic nces in the field of multimedia. Although the

human teachers. The result of bringing to multimedia devices and AI technology repr a significant improvement in the performance of the systems that can p more individualized instruction. This ada to individual students has been made p through the development of certain m within an ITM. A vital role is played by th *module* implementing the *user model* tailoring the system's behavior to the needs. User modeling aims to construct a in order to make predictions about behavior, so that the system can rec misconceptions and problems, identify and suggest suitable solutions. The user m makes hypotheses about user's conceptio reasoning strategies employed to achie current knowledge state.

It is apparent that the task of ob information about the user and in conclusions on the basis of what is supp very exacting, many interdisciplinary pro are to be tackled, and only partial solutio be attained. However, this fact underli importance of improving knowledge engin techniques to more effectively captur portray the user's knowledge. This module a crucial role in the overall operation performance of the entire system, ke running "snapshot" of the user and allow modeling problems can be found in [...silovsky, 1996].

...note that the learning process is ineherently ...deterministic and this circumstance induces ... necessity of managing uncertainty. The ...rtainty stems from several reasons, we just ...e ambiguity and multiplicity. The ambiguity is ...ed to the fact that user's errors are anyway ...onal mistakes and thus the same error can be ...ed, in different users, by different reasons. ... multiplicity, in turn, is related to the fact that ...rong behavior in order to solve a specific ...lem can depend on several misconceptions ...skill deficiencies.

... use of fuzzy logic represents an interesting ...ion to the problem of dealing with ...rtainty. In fact, in such way one can manage ...tatively different situations that the classical ...valued logic is unable to cope with. The basic ...ciples of fuzzy theory are presented in [Klir, ...] and several applications to teaching and ...ing problems are in [Liou and Wang, 1994, ...rice-Baumont and Derognat, 1994, Sawaragi ..., 1991, Hawkes *et al.*, 1990, Biswas, 1995].

... paper is organized as follows. Section 2 ...ls basic results of fuzzy theory. Next Section ...s how it is possible to represent user's ...itive states and psychological characteristics ...ugh suitable strings. Section 4 presents the ... features of a fuzzy algebraic structure that ...ges user representing strings. In Section 5 ... concept of fuzzy score is introduced in ...ion 6 it is shown that the fuzzy structure can ...tilized to monitor the evolution of the learning ...ess. Section 7 tackles the problem of ...uating the learning process and last Section ...hasizes the expressive power of the model as ...pared with other educational taxonomies

...he theory of fuzzy sets

...n a classical not-empty set U, a fuzzy subset μ_A: U ---> [0, 1]. The function $\mu_A(x)$ ... viewed as an extensione of the conc... characteristic function. Alternatively, a fu... can be denoted as follows: if A = {x_1, ... x_n} is a finite set and is represented as... $x_1+x_2+ ... +x_n$ of its elements, then

A = $\mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + ... + \mu_A(x_n$... A is infinite then A = $\int_U \mu_A(x)/x$, whe... symbol slash links each element of U ... membership grade. In both cases, a fuzz... on U is completely characterized by the ... (x, $\mu_A(x)$) where $x \varepsilon U$.

Let U={a, b, c} be a set of persons. S... that the individual **a** weighs 85 kilos, **b** ... 90 kilos and **c** 130 kilos. This is a possibl... set for fat people:

A = 0.49/a+0.53/b+0.85/c.

A fuzzy variable is a triple (Vf, U, R(... where Vf is the name of the variable, U... universe of discourse, u denotes the ... element of U and R(Vf; u) is a fuzzy su... U. For example "budget" could be the na... fuzzy variable where U=[0, ∞], and R(V... {(u, s(u) = 1), u ε [0, 100]} ∪ {(u, s(u) ... $((u-100)/200)^2]^{-1}$), u ε [100, ∞]}.

A linguistic variable is, in turn, a quintup... T, U, g, m) where V1 is the nam... variable. T is the set of values or linguisti... of V1, U is the universe of discourse, ... syntactic rule (a grammar) capable of gen... such linguistic values and m is a semant... that attaches to each element t of T its m... m(t), namely a fuzzy set on U. For examp... values of the linguistic variable "age" co... young, rather young, old, very old, and ... Each value is the name of a fuzzy varia... the universe of discourse U = [0, 100]. [2...

We note that adverbs such as "very", "... are called linguistic modifiers [Zadeh, ... because modify the meaning of terms s... "old" and "young" which are generators...

ions one gets the triangular functions, i.e. the gular symmetric numbers that are the fuzzy used to mathematize the terms of a linguistic ble [Pedrycz, 1994]. Lakoff has shown off, 1973] the necessity to define the terms of guistic variable beginning from the generators s. The following function is capable of rating the term set of a linguistic variable:

$= 2 * (x-a)/(b-a) * I_{[a, (a+b)/2]}(x) +$
$(x-a)/(a-b) * I_{[(a+b)/2, b]}(x),$
$U \subseteq R$, a, b ε U,
e $I[x_1, x_2](x) = 1$ if x ε $[x_1, x_2]$
wise 0.

e terms of the linguistic variable "evaluation" good, fairly good, not good, vemry good; rding to what suggested by [Schwartz, 1989], e following Sections we use for this variable a ence of values adjacent and uniformly buted, rather than an infinite sequence of es. If one wishes to use n terms for uation, the i-th term is the fuzzy set with bership function different from zero on the val [Max/n*i, Max/n*(i +1))/2], where x ε Max]⊆R and Max=highest possible rating. element of the variable "evaluation" can also denoted by the triple [a, c, b] where a+b)/2 and $\mu(x) = 0$, if $x \leq a$, = 1, $\mu(x) = 0$ if $x \geq b$ [9].

uzzy modeling of an ITM user

fuzzy sets can be used to model the vledge of a concept by a generic user. In fact, ntroducing the notions of linguistic variable fuzzy variable one can:

- del the inherent vagueness about the vledge of a concept
- oduce dynamically new terms (fuzzy labels) e linguistic variable, using suitable modifiers
- ieve a rich descriptive structure of the user's nitive, psychological and mental states

In fact, we propose that the user's cognitiv be represented by a string of the type:

$$a_n^{\alpha n} \; a_{n-1}^{\alpha n-1} \ldots a_1^{\alpha 1}$$

where the elements of the sets $\{a_i\}$ a didactic goals related to a specific navi within the ITM, whereas the elements {c fuzzy variables, namely values of a lin variable.

Let $\{C_i\}$ be the set of concepts related to of the knowledge domain present in th and let $\{O_i\}$ be the set of goals. We sa mastery of the concept Ci is very good system, interacting with the user, genera string $[O_{i1}, O_{i2}, . . ,O_{in}]^{vg}$ where the lab denotes the value "very good". In gen string such as the following represent singles out different learning levels for eac related to a specific concept:

$$[O_{i1},, , , O_{ik}]^{\alpha n}[O_{i(k+1)}, , , O_{i(k+2)}, . ,O_{ih}]^{\alpha n-1} \ldots [O_{i(h+1)}, , , O_{in}]^{\alpha 1}$$

where $\{\alpha_n\}$ are the values of the v evaluation.

Of course the user representation c enriched by taking into account other such as the psychological state. Such stat be evaluated using the linguistic va "attention" and "interest" associated to node n_i visited by the user during the nav within the ITM. In this case the element are the hypermedia nodes and the labels { values of the linguistic variables. For exa couple such as (node k, very interested) that the user, during the visit of the node been deeply involved. In this way the le style of a specific user can be represented linguistic variables such as the average le path or the average navigation time for stions, choice of a specific hypertextual link, pening on a concept, request of an example, mission of a topic already examined, access to ta base. Such actions can involve one or more es in the hypertext network and can be linked a specific semantics, i.e. each action can be rpreted in order to get information about user's nitive state. In such way the system can adapt behavior to user's needs.

following table summarizes the possible ons. To each element describing the model, "attached" the functions and variables therein ned.

itive state:
- -Right translation
- -Left transation
- -Comparison
- -Change in learning states
- -Distance between cognitive states

hological state:
- -Attention
- --Interest

base orientation:
- -Peak
- -Peak difference
- -Per cent right translation
- -Per cent left translation

ing styles:
- -Average length of deepening path
- -Average duration of node visits
- -Average duration of a tutoring session

e chosen the linguistic variable for evaluating states, the table shows the translations that ate changes in the values of the linguistic ble. Each architectural element is linked with ble functions and linguistic variables. Each istic variable and each function can be ted depending upon the type and strength of vidence that appears in a user action. In the el we limit our investigation to the variable uation" previously introduced.

of attributes used to classify the elemen We denote with $\alpha_1 \leq \alpha_2 \leq, , , \leq \alpha_n$ th numbers that represent the elements variable evaluation Vt. $a_{ij} = A_i^{-1}(\alpha_j)$ is th evaluated elements α_j with respect attribute A_i belonging to At. The string

$$a_n^{\alpha n} \quad a_{n-1}^{\alpha n-1} \quad \ldots a_1^{\alpha 1}$$

is the classification of U with respect attribute A_i [Gisolfi, 1992, Gisolfi an 1995, Gisolfi and Cicalese, 1996, Gisc Nunez, 1993, Di Lascio *et al.*, 1998 symbols a_i denote the *first parts* of the whereas α_n are the *second parts*.

Given the strings

$A = a_n^{\alpha n} \quad a_{n-1}^{\alpha n-1} \quad \ldots a_1^{\alpha 1}$

and $B = b_m^{\beta m} \quad b_{m-1}^{\beta m-1} \quad \ldots b_1^{\beta 1}$,

with $n \geq m$, the operation Π, commutati associative, associates with A and B the st

$$(a_n^{\alpha n} a_{n-1}^{\alpha n-1} \ldots a_1^{\alpha 1}) \Pi$$
$$(b_m^{\beta m} b_{n-1}^{\beta m-1} \ldots b_1^{\beta 1}) =$$
$$= c_{m+n-1}^{\gamma m+n-1} c_{m+n-2}^{\gamma m+n-2} \ldots c_1^{\gamma 1}$$

The operation Π splits in two operatio first is denoted by * and acts upon first pa second with • and acts on second one traditional subsets c_i and the fuzzy numb are obtained by applying the operators * a respectively.

The operation for the first parts is defi follows:

$$c_i = \begin{cases} \cup_{j=1..i} \quad a_{i-j+1} \cap b_j, & \text{if } 1 \leq i \leq m-1 \\ \cup_{j=1...m} \quad a_{i-j+1} \cap b_j, & \text{if } m \leq i \leq n-1 \\ \cup_{j=i-n+1...i} \quad a_{i-j+1} \cap b_j, & \text{if } n \leq i < n+ \end{cases}$$

where $\cup$ and $\cap$ are the usual set-th operations defined on the power set P(U). This operation has the following properties

Existence of the null element, namely U.

ne operation for the second parts, in turn, is fined as follows:

$n\alpha_{n-1} \cdots \alpha_1) \bullet (\beta_m \beta_{m-1} \cdots \beta_1) =$

$m+n-1\ \gamma_{m+n-2} \cdots \gamma_1$

here:

$$1/(k_1+k_2)\, d_i * \Sigma_{j=1\ldots i}\ d2_j * d1_{i-j+1} * (k_1 * \alpha_{i-j+1} + k_2 * \beta_j),\ 1 \le i \le m-1$$

$$= 1/(k_1+k_2)\, d_i * \Sigma_{j=1\ldots m}\ d2_j * d1_{i-j+1} * (k_1 * \alpha_{i-j+1} + k_2 * \beta_j),\ m \le i \le n-1$$

$$1/(k_1+k_2)\, d_i * \Sigma_{j=i-n+1\ldots m}\ d2_j * d1_{i-j+1} * (k_1 * \alpha_{i-j+1} + k_2 * \beta_j),\ n \le i \le m+n-1$$

nd the values d_i are obtained as

$_{j=1\ldots i}\ d1_j * d2_{i-j+1}$.

this operation the indices k_1 e k_2, respectively r A and B, and the arrays $d1_i (i=1..n)$ and $d2_j$ $=1..m)$, give the distributivity of the operation $\cap$ ith respect to $\cup$. The indices $d1_i$ and $d2_i$ epresent, respectively for the first and the second ring, the number of sets for which the union has een carried out in order to get the i-th class; the dices k_1 and k_2 denote the number of sets hose intersection has produced such sets.

The operation • is closed in the set numbers defined in [0,1], is as commutative and preserves the orderin fuzzy numbers.

Example

Consider the following matrix (concep that can be viewed as representativ cognitive state of an ITM user:

	C1	C2	C
O1	g	i	q
O2	i	g	g
O3	i	q	q
O4	s	s	q
O5	s	s	s

We can write:

$C_1 = [O_1]^g [-]^q [O_4, O_5]^s [O_2, O_3]^i$,

$C_2 = [O_2]^g [O_3]^q [O_4, O_5]^s [O_1]^i$,

$C_3 = [O_2]^g [O_1, O_3, O_4]^q [O_5]^s [-]^i$.

Let U = [0, 1] and suppose that

g(ood) = [0.8, 1, 1],

q(uite good) = [0.5, 0.7, 0.9],

s(ufficient) = [0.2, 0.4, 0.6],

i(nsufficient) = [0, 0, 0.2].

Multiplying the first parts:

$C_1 * C_2$

		[O1]	[-]	[O4,O5]	[O2,O3]
		[O2]	[O3]	[O4,O5]	[O1]
		[O1]	[-]	[-]	[-]
	[-]	[-]	[O4,O5]	[-]	

$C_1 * C_2 * C_3$

		[-]	[-]	[-]	[O1,O2]	[O3,O4,O5]	[-]	[-]
					[O2]	[O1,O3,O4]	[O5]	[-]
		[-]	[-]	[-]	[-]	[-]	[-]	[-]
	[-]	[-]	[-]	[-]	[O5]	[-]	[-]	
[-]	[-]	[-]	[O1]	[O3,O4]	[-]	[-]		
[-]	[-]	[O2]	[-]	[-]	[-]			
[-]	[-]	[O2]	[O1]	[O3,O4]	[O5]	[-]	[-]	[-]

s $C_1 * C_2 * C_3$ =
][-][][O2][O1][O3,O4][O5][-][-][-]

ltiplying the second parts

$= C_1 \bullet C_2 = (g, q, s, i) \bullet (g, q, s, i) =$
$h_6, h_5, h_4, h_3, h_2, h_1)$
note that: $d_{C1} = d_{C2} = (1, 1, 1, 1, 1, 1, 1)$;
$_1 = K_{C2} = 1$;
$= 1, d_{H2} = 2, d_{H3} = 3; d_{H4} = 4$;
$_5 = 3; d_{H6} = 2; d_{H7} = 1$
$_H = 2$, because the string H represents the
duct of two strings:
$= 1/2\ (i+i) = [0, 0, 0.2]$
$= 1/4\ (i+s+i+s) = [0.1, 0.2, 0.4]$
$= 1/6\ (i+d+s+s+d+i) = [0.233, 0.366, 0.5]$
$= 1/8\ (i+b+s+d+d+s+b+i) =$
$= [0.375, 0.525, 0.625]$
$= 1/6\ (s+b+d+d+b+s) = [0.5, 0.7, 0.833]$
$= 1/4\ (d+b+b+d) = [0.65, 0.85, 0.95]$
$= 1/2\ (b+b) = [0.8, 1, 1]$.

w we carry out the multiplication
$(b, d, s, i) = (\delta_{10}, \delta_9, \delta_8, \delta_7, \delta_6, \delta_5, \delta_4, \delta_3,$
$\delta_1)$ and we get for the not empty first parts:
$, \delta_6, \delta_5, \delta_4) = ([0.510, 0.690, 0.810],$
$417, 0.583, 0.733]\ [0.325, 0.475, 0.642]$
$240, 0.360, 0.540])$.
applying the procedure for linguistic
roximation described in [Gisolfi and Nunez,

[0.417, 0.583, 0.733] ---> IB(s,q),
included between "s" and "q"
[0.325, 0.475, 0.642] ---> NT(s), name
to "s"
[0.240, 0.360, 0.540] ---> s
The final result is $C_1 * C_2 *$
$= [O_2]^q [O_1]^{IB(s,q)} [O_3, O_4]^{NT(s)} [O_5]^s$.

5. From fuzzy scores to linguistic terms

Let $t[a_1, c, a_2]$ and $t_1[b_1, d, b_2]$ be tw
sets whose membership functions are eit
s or π. In both cases two paramet
sufficient to single out univocally the fu
The *center* of the shell of a fuzzy set
real number $m_t = 0.5*(a+b)$.
The function

$$\Omega(t[a_1, c, a_2], t_1[b_1, d, b_2]) = ((a_1+a_2-(b_1+b_2))/(a_2-a_1+b_2-b_1)$$

is said *superposition grade* of two fuz
and t_1 [Hellendoorn, 1992]. It is worth
that: if $\Omega = 0$, then the center of the
the fuzzy sets is identical; if $\Omega > 0$, the
m_{t1}; if $\Omega < 0$, then $m_t < m_{t1}$. Moreover
1, then we can adfirm that $m_t >> m_{t1}$, wh

neans that the evaluation expressed by t is more avorable than that expressed by t_1.

et U=[0, Max] be the interval of possible scores. A *fuzzy score* is a fuzzy number FS: U ---> [0,] defined as follows:

$$
FS(x) = \begin{cases} l(x) & \text{if } x \in [0, a] \\ 1 & \text{if } x \in [a, b] \\ r(x) & \text{if } x \in [b, Max] \end{cases}
$$

where l(x) is a function defined in U and whose ange is [0, 1], increasing, left continuous and uch that l(x)=0 for $x \in [0, \lambda_1] \subseteq [0, a]$ whereas (x) is also defined in U and ranges in [0, 1], but s decreasing, left continuous and such that r(x) =) for $x \in [\lambda_2, Max] \subseteq [b, Max]$. A class of unctions satisfying such conditions are just the above mentioned functions S and π. In this way each element of Vt is the linguistic translation of a fuzzy score.

t is easy, given a fuzzy score, to find the corresponding term of Vt. Let p be a FS, if $|\Omega(p, t_0)| = \min_{t \in Vt} |\Omega(p, t)|$, then t_0 is the term of the linguistic variable Vt corresponding to the fuzzy score.

Example

U = [0, 10];

Vt={insufficient, sufficient, quite good, good, very good};

insufficient = [0, 0, 10/5]

sufficient = [10/5, 0.5*(10/5+10/5*2), 10/5*2

quite good = [10/5*2, 0.5*(10/5*2+10/5*3), 10/5*3]

good = [10/5*3, 0.5*(10/5*3+10/5*4), 10/5*4]

very good = [10/5*4, 10, 10]

Give FS = p(u; 4; 6), one gets:

$|\Omega(p, \text{insufficient})| = |(10\text{-}2)/(2+2)|$

If FS = p(u; 2; 3), one has:

$|\Omega(p, \text{insufficient})| = |(5\text{-}2)/(1+2)|$

$|\Omega(p, \text{sufficient})| = |(5\text{-}6)/(1+2)|$

$|\Omega(p, \text{quite good})| = |(5\text{-}10)/(1+2)$

$|\Omega(p, \text{good})| = |(5\text{-}14)/(1+2)|$

$|\Omega(p, \text{very good})| = |(5\text{-}18)/(1+2)$

and we get FS = sufficient

For the sake of simplicity, withou generality, we use for the fuzzy sco functions S and π, in such way a fuzzy be expressed either giving directly fuzzy number or considering a set of FS(x)). We denote this choice by example, the fuzzy set

0/0+0/20+0.1/40+0.5/50+0.6/60+0.9/

if we take seven points and Max = example of discretized fuzzy set. I case, in order to translate the fuzzy into a term of the variable Vt, it is s compute the minimum of the functio In the second case, the couples of va interpolated, and and the problem ca by applying least squares method fi and then for r(x). These seco polynomial functions allow to sing Zadeh's function represented by the way we are lead to the first case. W improve the linguistic approximation suitable linguistic modifiers to the set

6. The assessment

The first column of the following ta the goals related to a specific concept

O_1	y_{11}	y_{12}	...	...	y_{1n}	
...	...	...	...	...	...	
O_i	y_{i1}	...	...	...	y_{1in}	

he ITM expresses for each O_i the fuzzy score, where y_{ij} j belongs to the interval [0,1]. $s(x_i) = y_{ij}$ neans that the assessment, which evaluates equal o x_i% the achievement of the goal, is assigned a redibility level equal to y_{ij} [3]. A possible range f values for x_i is the following: 0%, 20%, 40%, 0%, 60%, 80%, 100%. For the i-th row the set f couples (x_i, $\mu_{[0,1]}(x_i)=y_{ij}$) represents a fuzzy core (FS_Z) and thus such set should comply vith the above definition of fuzzy score (FS_Z). In urn, if the score is expressed by the fuzzy umber [a,c,b] it means that we are sure that the oal cannot be achieved with a confidence level igher than b% and lower that a%, whereas this onviction for other values is expressed by the unction l(x) and r(x) introduced in Section 5.

et us considere the i-th row of the previous table. he values $\{y_{ij}\}$ single out either a fuzzy set t or number S or π according to the choice made. It possible to locate two values t_1 and t_2 of the nguistic variable Vt

uch that $t_1 \le t \le t_2$. They can be computed by neans of the Hellendoorn's function Ω:

$_2$ = min $|\Omega(t, Vt)|$

= min $|\Omega(t, V')|$, where $V' = Vt \setminus \{t_2\}$.

lowever, in general t is different from t_2. In such ase the above mentioned procedure of linguistic pproximation [12] allows to single out the nguistic modifier and consequently the value of e variable Vt related to each row O_i in the ble. These values are reported in the last column E_i. By using a larger subset of Vt the problem finding the element of Vt which best represents e linguistic translation of the values y_{ij} can be ckled. The overall evaluation can be computed follows:

$= E_1 \bullet E_2 \bullet \ldots \bullet E_i \bullet \ldots \bullet E_m$.

Vith each fuzzy number E gets associated a nguistic term. However it is also possible to state overall assessment about the user. In fact, let

The product $A = \Pi A_k$ gives a evaluation of the user described by ta kind reported above.

7. Monitoring learning process

Let {C1, C2, , , Cn} be the set of co can be grasped by the user during session and suppose that the system each concept C_j a string whose first p the conditions $O_i = C_j^{-1}(\alpha_i)$, w quantities α_i are the values of th "evaluation" as defined in Section 3. way we have in different moments matrices (concept, goal). The set matrices describes the temporal evolut user's learning process. At time t = representation of the cognitive state is the k-th matrix:

	O_1	O_2	.....	O_h
C_1	α_{11}	α_{12}	.....	α_{1h}
C_2	α_{21}	α_{22}	.....	α_{2h}
.....			.....	
C_n	α_{n1}	α_{n2}	.....	α_{nh}

For each matrix it is possible to carr products among rows:

$C(t=t_1)=C_1(t=t_1)\ \Pi \ldots \Pi\ C_n(t=t_1)$, ma

$C(t=t_2)=C_1(t=t_2)\ \Pi \ldots \Pi\ C_n(t=t_2)$, ma

$C(t=t_k)=C_1(t=t_k)\ \Pi \ldots \Pi\ C_n(t=t_k)$, ma

Each product gives a classification overall evaluation referred to the in product is carried out. In turn, the prod

$$C = \Pi_i\, C(t=t_i)$$

gives rise to a string that represents th cognitive state of the user.

By multiplying rows belonging to matrices, one can get informatio cognitive levels for each concept "o

$C^2 = C_2(t=t_1) \Pi ..... \Pi C_2(t=t_k)$;

$C^n = C_n(t=t_1) \Pi ..... \Pi C_n(t=t_k)$;

Every C^i represents the user's learning level of the concept C_i. As every string is constructed at different instants, the product represents the final state of the cognitive evolution for a specific concept. While the previous string C gives the final evaluation for each didactic goal beginning from the representation of the conceptualization level achieved, the new product:

$$C^+ = \Pi_i C^i$$

emphasizes the temporal evolution of the learning process.

It is also possible to build strings relating goals and concepts. We note that in mastery learning the tutoring strategy [Block, 1971] aims at give students mastery about the basic elements of knowledge. By linking knowledge with concepts and abilities with goals a string, in which $C_i = O_j^{-1}(\alpha_i)$, evaluates the mastery level of each ability related to concepts. Thus a string such as the following:

$O = [C_{i1}, , , C_{ik}]^{\alpha n}[C_{i(k+1)}, , , C_{i(k+2)}, , , C_{ih}]^{\alpha n-1} ... [C_{i(h+1)}, , , C_{in}]^{\alpha 1}$

adfirms that the concepts $[C_{i1}, , , C_{ik}]$, expressed by mastery O, have been achieved with evaluation α_n. Such string gives us a computable schema to tackle the problem. For example, the ability to solve a polynomial equation is related both to the concept of zero of a polynomial and to the notion of algebraic structure on which depends the solubility of an equation. A string such as the previous one offers information about the way mastery levels of these concepts contribute to the overall ability. Thus by multiplying the columns belonging to the same matrix one gets the following string:

$O(t=t_1)=O_1(t=t_1) \Pi .... \Pi O_n(t=t_1)$, matrix 1;
$O(t=t_2)=O_1(t=t_2) \Pi .... \Pi O_n(t=t_2)$, matrix 2;

$O(t=t_k)=O_1(t=t_k) \Pi .... \Pi O_n(t=t_k)$, matrix k;

and thus $O = \Pi_i O(t=t_i)$, and a final label expressing the contribute of each concept to the overall mastery is attached to each concept.
Moreover the product of columns belonging to different matrices:

$O^1 = O_1(t=t_1) \Pi ..... \Pi O_1(t=t_k)$;
$O^2 = O_2(t=t_1) \Pi ..... \Pi O_2(t=t_k)$;

$O^n = O_n(t=t_k) \Pi ..... \Pi O_n(t=t_k)$;

expresses the overall contribution, in the interval $[t_1, t_1]$, of each concept to the goal taken into account. Thus in different strings the same concept can appear with different labels. The final product:

$$O^+ = \Pi_i O^i$$

which permits to attach, at the end of tutoring sessions, one evaluation label to each concept.

8. Concluding remarks

Our approach to user modeling for ITMs allows to represent cognitive states, learning styles, psychological states, mastery levels. The fuzzy strings that are our basic elements are able to express concepts and goals presented in the taxonomies well known in the literature [De Landesheere, 1990]. The algebraic structure dealing with such strings allows to monitor and evaluate all the elements present in the learning process of an ITM user and gives us a fuzzy calculus on a set of words. We note that this model of evaluation can be used both in an ITM and during tutoring activity. In this second case the degree of subjectivity during user's assessment can be reduced so that the overall evaluation can become more objective.

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