SIGNAL ANALYSIS: AN APPLICATION OF C-MATRIX

A. Gisolfi, S. Vitulano

Istituto di scienze dell'informazione
Facoltà di scienze - Università di Salerno
Salerno
ITALY

A Calculus of Partition

We propose to present some application to Pattern recognition of a " matematical game " started some years ago (1), which was named " C-calculus " for reasons which will be reminded in the sequel. We wish to state forthwith that it is simpler in principle then ordinary aritmetics; varius fields can be envisaged in which it might prove to use: e. g. maninfold topology, integration theory, fuzzy sets (where it might provide a natural tool for numerical computation), measure theory in phisics, data bases structures, neural models, etc.

We begin therefore by reminding the game with which it all started. Take any integer positive numbers, and apply to them the rules of aritmetics, with the restrictions that only the direct operations, sum and multiplication, be allowed, the invers ones, substraction and division, forbidden; define furthermore the sum and the product of any two digit as follows

1) $$a + b = \max (a, b)$$
$$a \times b = \min (a, b).$$

It would be an easy matter to demonstrate that, provided the " single digit operation " 1) are meaningful, one can operate in the same way on objects (subtraction and division being of course barred), such us vectors, matrices, etc., obtaining additivity and commutativity whenever they hold in aritmetics.

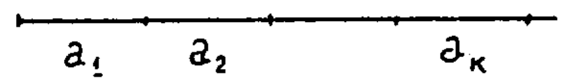
These " numbers " or " strings of digits ", with the operation 1), form clearly a commutative semi-ring. As in aritmetics each " digit " plays two different roles: one intrinsic to it (cardinality), the other (position) relative to the string in which it belongs. The next remark is that standard set theory treats only intrinsic properties of sets. If in 1) we interpret

+ as union " $\cup$ " and x as intersection " $\cap$ ", we can immediately transport all that was said thus for " strings of sets ", or " composite sets ", " C-set " for short.

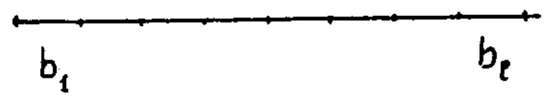
Operating on C-set as before, with $\cup$ and $\cap$ in place of + and x in 1) (a, b denote now the " simple " sets of which C-sets are strings), one has " C-calculus ": a commutative semi-ring which permits, from same given C-sets, to generate any number.

An example of C-operation of special relevance for our present purpose is the following:

Consider a segment S partitioned in segments $a_1, a_2, \dots, a_k$; this partition $A = a_1, a_2, \dots, a_k$



Consider now the same segment partitioned in a different way $B = b_1 \, b_2 \dots b_l$



Consider now A and B as C-sets: the element of each partition, or string, are " simple " sets; C-multiplication of A and B gives

$$A \times B = B \times A = a_1 \, a_2 \dots a_k \times b_1 \, b_2 \dots b_l =$$

$$= c_1 \, c_2 \dots c_p = C$$

and it is immediate to verify that the simple sets of the product are obtained, in order, by joining on the segments the terminal points of both partitions A and B.

The C-product of two partitions gives thus the refinement of one by the other: C-calculus is the natural way of composing partitions, or coverings. In fact, the same property holds true in any number of dimensions (2). This is the key property of C-calculus as regards its application to pattern recognition.

Under " suitable " circumstances (to be defined explicitly in the sequel) this procedure can be carried through to the extreme limit of perfect reconstruction of the original picture (as digitized at the finest possible level: e. g. with a $2^{10} \times 2^{10}$ grid for the original, it may be reconstructed by covering it stepwise with, say, a $2^3 \times 2^3$ grid). Duriung this process many things which one does with specific tecniques,such as contour extraction, contrast enhancement, feature extraction, etc., can be performed by interpolating in it steps with " answer " questions of this sort and become part of the algorithm.

But the application of C-calculus will often fail; the original image may not be thus reconstructed. There is an element to be considered, which was before ignored through the adjective " suitable ": the size of the window. It is a feature of our approach that the critical size, below which total reconstruction of the picture is inpossible, is determined by the structure of the picture itself, and is not a matter of guesswork or trial and error.

One can arrange readings, and ways of analizing them, from grids having sizes appropriates to constitute in fact filters that see some wanted features and are blind to others.(3, 4, 5,)

C-MATRIX

1.- Consider a pattern F in any number N of dimensions (for the sake of illustration we restrict here N = one or two), and consider only one of some K " attributes " of interest, e. g. the level of greyness (which we may suppose now digitized). An N-dimensional grid of (sqare) windows generates, as we have seen, a C-set in N + 1 (K = 1) space: C-multiplication of all C-sets obtained by displacing (according to some rule)the grid over the pattern yields, after a suitable number of iterations, a pattern F, which coincide with F if condition 2) is respected, differ from F otherwise. This process we have called C-filter; as such, it might deserve per se mathematical investigation. Our interest here is rather with concrete ways of exploiting the typical new feature of C-calculus, that of permitting either " precise " measurement of a whole through serial composition (C-multiplication of " coarser " partial parallel measurements) (each, a C-set from the grid), or " filtering operations ". " Precise " and " coarse " are to be understood as in physics. We have found profitable, for this purpose, to introduce a mathematical object which (for any N, K = 1) is always 2-dimensional: the C-matrix.

2.- We define the C-matrix. Its element c_{hk} has as row label h (= 2, 3, ...) the linear size of the window w_h of the scan-

ning grid G_h (C-calculus will be more useful the larger can be kept the minimum h needed for a given analisis); the column label k (= 0, 1, 2, ...) denotes the possible values of the dinamic: K = M - m as read through w_h. The grid G_h scans the pattern according to some criterion (in two dimensions, the best has proved to run down the main diagonal) by one step at a time (" 1 " means the original digitizing window, if any, or simply the " resolving power " of the sistem); G_h has thus h (= h, square grid h, linear) scanning positions; any value k of the dynamic is registered . (= 0, 1, ...) times in the windows, for each position i = 1, ..., h of the grid G_h. In conclusion

The building of C-matrix seems, from the definition just given, a much more imposing task than actually is. In fact, the operations that lead to c_{hk} are performed in parallel for k = 0, 1, ... so that the C-matrix is built one row at a time, or in one piece with suitable hardwere; also, the procedure has to be started from the bottom row, i. e. largest w_h, and it becomes an obvious matter to implement the algorithm with devices that suppress the need to explore, moving towards smaller h, territories where, e. g. k = 0 (uniform greyness) for a larger h. Nor need one proceed through the full sequence h_{max}, $h_{max} - 1$, ... , 3, 2: large jumps may be made.

An element c_{ij} of C-matrix tell us thus: if vanishes, that the value j of the dynamic of the pattern can never be seen through windows of size w_i; if not, how many times that dynamics seen by no matter which window w_i.

It is easy then to read from the C-matrix many features of the pattern; e. g. if only $c_{i,o} = 0$, all i, the pattern is a plateaux, of constant grayness throughout; the max i for which $c_{i,o} = 0$ is the width of the largest plateau; a highest-slope of (1-dimensional) signals shows as the sequel $c_{ij} = 0$, j max, for each i.

3.- Contour extraction

This is a relevant step in every problem of P.R., with a wide literarture and a large number of tecniques available: gradients and threshold, thinning, joining or separating of broken or interpetrating pieces, template matching, etc. (6, 7, 8,). We can extract contours with the following rule. Consider

the first among the columns of the C-matrix, let it be the column h, in which the elements c_{ij} go through a maximum, at some value i. Take the corresponding window w_i as an element of the grid of a C-filter, given by C-multiplication of C-sets whose simple sets vanish if smaller than the (hyper) rettangles w_i x h (i. e. h is a threshold on dynamic).
Fig.'s 1, 2 illustrate the procedure.

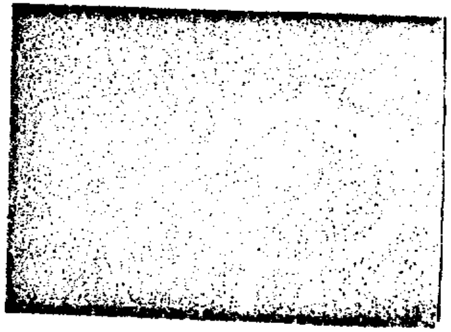


fig. 1
I.Matrix

fig. 2
O.Matrix

Comparison with ather algoritms (10, 6, 11) was made; it appears that thresholds come out of the C-matrix and do not have to be guessed, and that computation time is here less than with RAG, LAG, PT.

TEXTURES.

The analysis of textures, both per se and as backgrounds, is almost a science within a science; see, e.g. Gibson (9), Koehler (10), Haralik (11).

Our views on this subject do not belong in this short summary of our work; more than " percepta " are " phenomena ", in the sense of Kant, quantum mechanics or the Vedas, to which the " noumenon " and the " observer " equally concour. We wish only to report here, in conclusion, that C-calculus has proved especially " natural " in this context, leading to ready and elementary classification, analysis and discrimination.

We refer the reader to previous works (16), where some examples are reported. It is the " chessboard and saucer " game we were mentioning earlier, which is easily implemented through C-calculus into algoritmic simulations which may be

rather close in principle to the actual operation of neuronal tissues. (12, 13)

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