# The spontaneous geometry : a microworld for the primary school

Theme 4 : Microworlds

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ABSTRACT

The child's conception of geometry was the object of this LOGO microworld developed for children of the primary school. The methodological background of the work was the "Piagetian learning" and we explored the "intelligent" features of computer programs for their potential in instructional interventions and, at the same time, we validated the lying behind theory.

We note that this work is a methodological one: in fact we illustrate the structure of the microworld and how this learning environment was developed. The spontaneous geometry includes some experiments into the processes by which children come to understand the basic principles of geometry. These experiments were immersed into a suitable microworld which was structured in seven lessons; problems of change of position were first examined, then we deal with the conservation and measurement of length. Finally we discuss some properties of triangles and polygons.

## 1. INTRODUCTION

The purpose of this study was to present a LOGO microworld devised to explore the learning of spontaneous geometry by children. According to Piagetian view of learning processes we think that the interaction between the child and the object of study is crucial in order to get a correct learning.

The LOGO language was developed for the purpose of enhancing thinking processes and procedural thinking skills with the use of technology. LOGO is highly motivational and offers several levels of challenge, making it a profitable learning environment for wide ranges of age and ability. It provides a good opportunity to get a clearer understanding of the interactive nature of the teaching-learning processes.

## 2. THE SPONTANEOUS GEOMETRY

Piaget's investigations about the development of intelligence in children locates three different stages:

1) during the first eighteen months of life: level of sensori-motor intelligence

2)below the age of 12: level of concrete operations

2a)between 2 and 6: pre-operational level

2b)between 6 and 12: operational level

3)between 12 and 14: stage of formal operations

The investigation reported in the present paper refers to the stages 2b.

The study of how children come to measure is very interesting because the operations involved in measurement are on one hand concrete (visual estimates of size, etc.) and on the other are quite complex so that children cannot fully elaborate them until the age of 9-11.





Moreover, it is important to avoid making any use of the notions acquired by children in the course of their formal education: otherwise we would obscure important psychological findings.

## 3. THE STRUCTURE OF THE MICROWORLD.

A "microworld" could be defined as an investigation field related to the computer with which one can interact and whose features spontaneously awake child's interest by involving him both emotionally and culturally.

In order to construct an adequate microworld we have to include some basic symbols and terms apt to help the growth of the system. In other words, a microworld furnishes a dynamic semantics for a formal system.

A crucial point is the presence in the system of suitable operations by which the child can build new objects and operations always operating in an interactive environment.

We developed our microworld with the aim of overcoming the usual problems facing the learning of the basic geometrical properties (which may have caused headaches to some of us in our schooldays); the approach aims at unifying some basic results about child's conception of geometry into a suitable computer-aided system.

The microworld is organized in seven lessons; each of these can be selected in the main menu (Fig. 1).

In the following we shall illustrate some of the most interesting features of the lessons.

### 3.1. LESSON 1. Changes of position.

The lesson explores the concept of "change of position" which is fundamental for Euclidean metrics. The study of the growth of Euclidean metrics in children leads to two critical points. The first of these is that there is a close relationship between the child's growing ability to describe changes of position and his gradual construction of adequate systems of reference. The link between groups of changes of position and coordinate systems is of invaluable importance for the following lesson, because of the circumstance that no measurement can exist without a well-defined group of positional changes.

The second crucial conclusion is that the growth of knowledge is not a matter of simple accumulation. Children de-centre relations which they have learnt and transform the latter by combining them with other relations. The elaboration of children's concepts to the point where changes of position give way to grouping and lead to a coordinate system is a reversal of these concepts, for they began with a egocentric attitude to relations regarding space and measurement. It is worth noting that the child carries out the changes of position utterly disregarding the path from the starting point to the other extreme.

The lesson first shows the turtle surrounded by well-known places (school, home, park, etc.) (Fig. 2) and asks the child to move the turtle towards the buildings. As a second step the child can locate in a different way the buildings so that the model can be closer to his actual life.

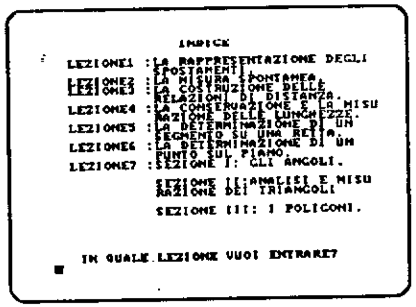



fig.1

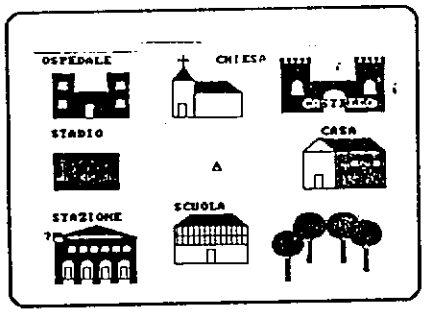



fig.2





There is a strong psychological link between the ability of children to construct a map of their neighbourhood and thei attitude to draw a path in the map.

### 3.2. LESSON 2. Spontaneous measurement.

The ruler we are accustomed with is the result of many operations unknown to the child and, as a consequence, it is useless to him. It is instead interesting to investigate how children construct their own rulers and units of measurement. We note that the problem of measurement of an object is closely related to that of conservation of the length of another object whose length will not be altered by its movement.

In the microworld, some strokes are first shown, in several positions, and the child is asked to judge whether they are equal or not. Subsequently the subject is shown a tower made of blocks and he is asked to build a similar tower (Fig. 3). The child has at his disposal blocks of different size and color. The microworld then asks the subject to utilize only blocks of the same size (Fig. 4) and finally the child is asked to build a block and verify how many blocks are necessary to make the tower. The concept of unit of measurement is thus acquired by the child in an operational way by means of the notions of partition and change of position.

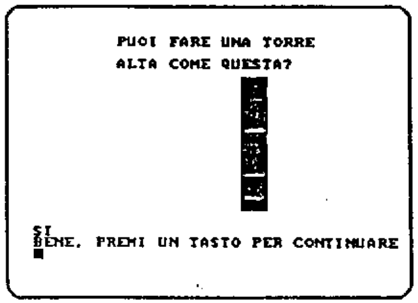



fig.3

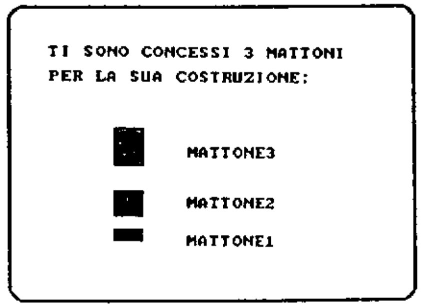



fig.4

3.3. LESSON 3. Reconstructing relations of distance.

This lesson is devoted to study of how children judge distances. We use the term "distance" to refer to the linear separation of objects, i.e. to empty space, and the term "length" to refer to objects, i.e. to denote the size of filled space. It is interesting to see how children realize the difference between the two concepts; furthermore it is worth examining how children reconstruct the space as a whole. The child first realizes elementary topological relations and gradually achieves the concept of Euclidean space. All things considered, the notion of distance is therefore vital in order to achieve the concept of coordinate system and to build a metrics in the space.

The starting point of the lesson shows three points on a straight line and the subject is asked to judge their distances. Then two trees are shown to the child and he is asked to state whether they are "near" or "far apart". A stroke is placed between the trees and the child is again asked whether they are still as "near" or as "far apart" (Fig. 5). Many slight variations on the basic theme are proposed by the microworld: trees are moved, several screens can be placed, windows in the scree can be present, etc.

The question of symmetry is raised in a similar fashion (Fig. 6) and

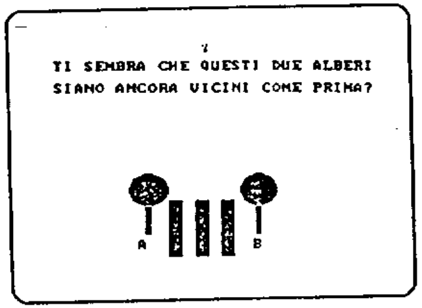



fig.5


fig.6





because of the fact that the identity AB = BA does not appear self-evident the child is gradually lead to the correct understanding of the symmetrical character of distance.

3.4. LESSON 4. Conservation of length.

The concept that the length of an object remains constant throughout any change in position is essential to the construction of a metric space. The conservation of distance yields an important clue but the problem now is to investigate what happens when objects are moved. The answer lies in the conservation of length when objects change their positions. In fact, for each newly filled site there is a corresponding site which is newly emply and vice versa which implies the conservation both of the distance between objects and of the length of objects when moved.

The microworld first shows a straight line and a snake-like one and the child is asked to judge their length (Fig. 7). Then the "snake" is straightned out and again the subject is asked the same question. Some variations on the theme are shown and finally the length of two strokes coincident with their extremities if compared. After staggering (Fig. 8) children do not take account of both ends and judge their length as different.

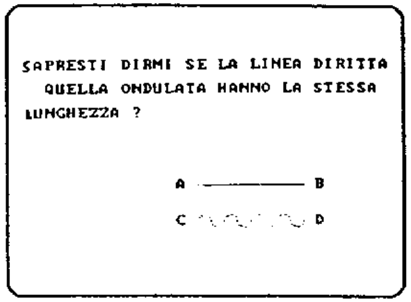



fig.7

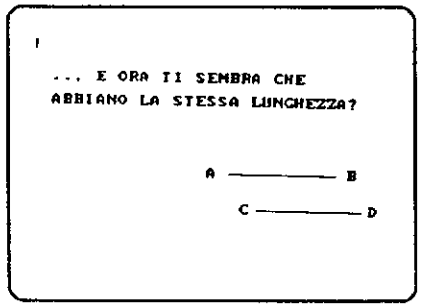



fig.8

Subsequently our concern is to analyze the intimate nature of the metrical operations and their relation to the conservation of length. The foregoing analysis of spontaneout measurement showed that operations of measuring are born from subdivision (choice of a unith-length) and change of position (use of a unit-length). Measurement if therefore to be studied as a synthesis of the latter operations.

Children are first presented two equal strips (Fig. 9). One of the strips is then cut in several parts and the question is again whether the two strips are still the same length. The child is then asked to judge the length of strips of different lengths (Fig. 10).

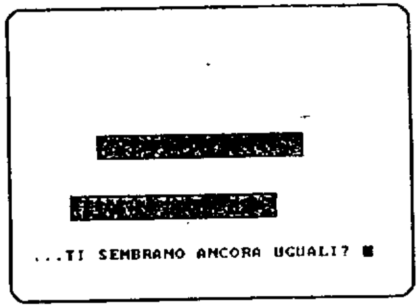



fig.9

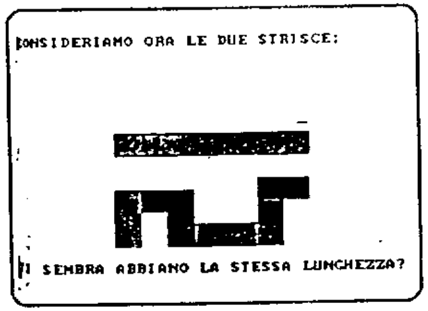



fig.10

3.5. LESSON 5. Subdividing a straight line.

In this lesson children are asked to locate a particular segment on a straight line using several measuring techniques. Given two straight lines AB and CD the microworld asks the child to find a point F along CD to correspond with a point E on AB such that the segment CF equals AE (Fig. 11). Our concern is the development of measurement as such and, in order to accomplish this goal, the subject is then asked to judge how many





segments CF are contained in AB (Fig.12) and eventually the concept of metric unit is achieved.

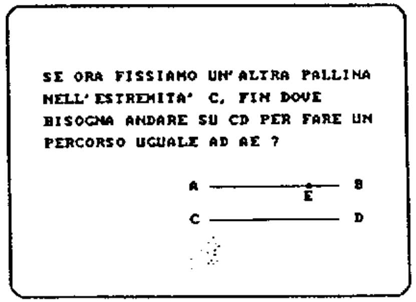



fig.11

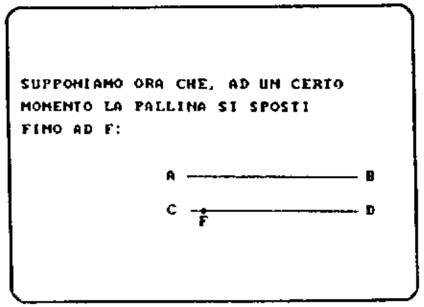



fig.12

The construction of a metric unit thus involves a twofold generalization. The measurement of length and distances is closely related to the circumstance that any length can be partitioned into a series of segments. Subdivision therefore enables the child to think of a unit as forming part of a whole, i.e. an elementary common part. On the other hand, subdivision cannot be attained without change of position: in fact, subdivision implies that the unit might be applied indefinitely and thus the concept of unit involves both the operations of change of position and subdivision.

3.6. LESSON 6. Locating a point in a plane.

The microworld then explores the two-dimensional metrics and the child is first faced with the probles of locating a point in a plane. In fact, there is a preliminary enquiry as to the way in which children set about measuring straight lines forming angles: the answer is that the measurement in two dimensions is carried out by children by means of paired measurements along the two axes of a right angle. Furthermore, children cannot locate a point in two-dimensional space without first developing a coordinate system.

The microworld presents two squares to the child and asks him to locate in S2 the dot already present in S1 (Fig. 13).

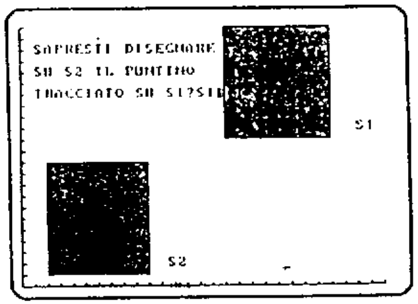



fig.13

A coordinate system is present in order to make the problem easier. Some variations on the theme are then shown and it is easy to realize that measurement in two dimensions presents the same difficulties (further complicated) as the synthesis of subdivision and changes of position referred to the line.

3.7. LESSON 7. Angles, triangles and polygons.

The last lesson is the most comprehensive and it was divided into three sections. The first deals with the measurement of angles. The second with the measurement of triangles and the last is devoted to the study of the properties of some polygons.



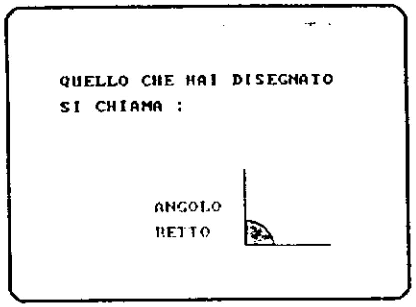


3.7.1. Section 1. Angular measurement.

Before children can locate a point in a plane, they have to develop a system of one-one correspondences with axes perpendicular. Rectangular coordination thus is closely related to the principle of one-one correspondence. On the other hand, angular measurement depends on the principle of one-many correspondence.

The child is first faced with the problem of drawing a right angle (Fig. 14) and then he is shown a drawing of two supplementary angles and is asked to make another drawing exactly similar (Fig. 15).

Many other questions about properties of angles are then discussed in the microworld and the child gradually understands the meaning of angular measurement.

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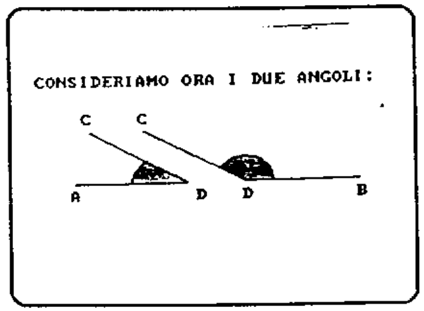



fig.15

3.7.2. Section 2. Measurement of triangles.

The cognitive process for the measurement of triangles is similar the that sketched for the angles: the child has the same difficulty in mentally decomposing and recomposing the distances involved.

The child is first asked to draw various tupes of triangles (right-angled, equilateral, scalene, etc.) (Fig.16).

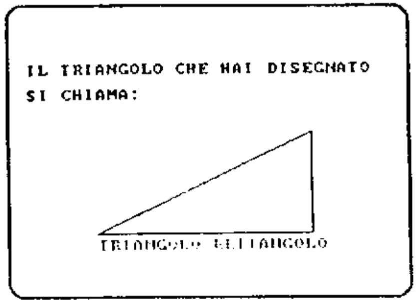



fig.16

We then focus our attention on the relationship between the well-known property of the angle of a triangle, i.e. their sum is always 180 degrees, and the way children gradually discover this property. The core of the problem is the "deductive composition" which allows to add the measure of the angles and then to infer the generalized law.

The child is first shown a right-angled triangle whose angles are then "cut" and juxtaposed (Fig. 17) and is simply asked how many degrees is the resulting angle (Fig. 18).

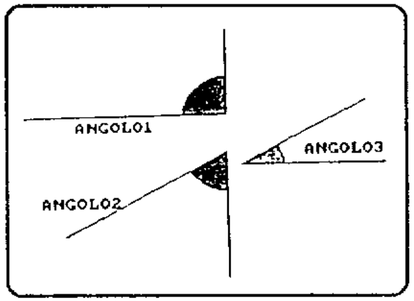




fig.17

fig.18

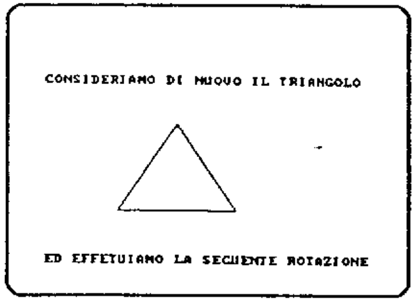




The experience is repeated with differently shaped triangles: it shows how a sequence of empirical actions leads to a necessary conclusion having a generalized validity.

3.7.3. Section 3. Polygons.

The microworld finally discusses some properties of polygons. The rotation of a triangle in order to get a regular polygon is the operational way which allows the child to construct many polygons (Fig. 19-20). He learns in such way that the circle is the "limiting" obtainable polygon.



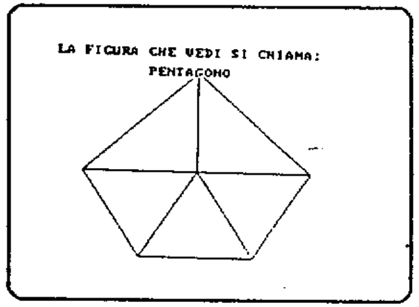




fig.19

fig.20

The microworld subsequently presents an irregular polygon (decomposable into triangles) (Fig. 21) and the child gradually discovers that the sum of the interior angles in a polygon if obtained by multiplying the number of juxtaposed triangles by the sum of the interior angles, minus the sum of the centre angles of the polygon (Fig. 22).

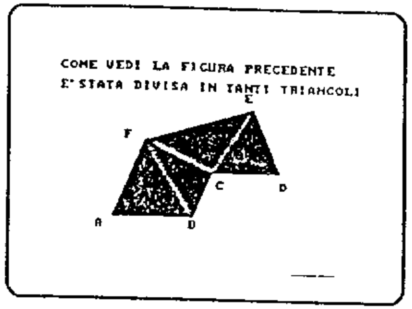



fig.21

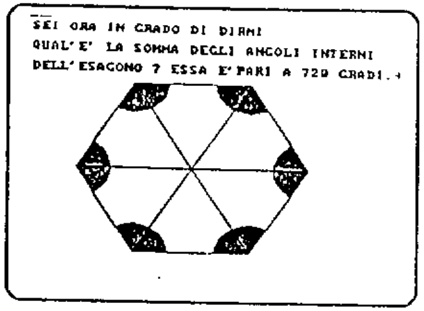



fig.22

Also in this case we have first an empirical procedural behavior which eventually achieves a logical deduction having the required characteristics of generality

## 4. CONCLUDING REMARKS

It is our opinion that self-guided discovery need to be mediated within a suitable instructional context. The fundamentals of primary school geometry may be easily made accessible to children in presence of a new class of intelligent tools that combine powerful performance systems and instructional aids.

In our microworld the tools are a set of symbolic manipulation LOGO programs for performing some interesting problems of spontaneous geometry. The LOGO experience was certainly an enjoyable one for the children who began to learn about how they think. It was apparent from observing the enthusiasm with which children interacted with our microworld that children of ages 8-10 can benefit from this type of computer experience. In conclusion, this research provides significant evidence of fruitful LOGO experience of geometry-learning ability at primary school level.





## REFERENCES

1) Piaget J., Inhelder B., Szeminka A.: "The child's conception of geometry", The International Library of Psychology, London, Routledge and Kegan (1960)
2) Piaget J.,Inhelder B.: "The child's conception of space" The International library of Psychology, London.
3) Gisolfi A. and Vitulano S: "Apprendere ad apprendere", Didamatica 88, Milano (1988)
4) Gisolfi A.: "Un ambiente per l'apprendimento: costruzione di un micromondo di italiano", Didamatica 89, Bari (1989)
5) Bork A.: "Learning with computers", Digital Press
6) Kearseley G.: "Artificial intelligence and education", Addison Wesley
7) COMPUTERS IN EDUCATION. K.Ducan and D.Harris eds. North-Holland 1985.