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A TECHNIQUE FOR TEXTURE ANALYSIS USING C-CALCULUS

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Zusammenfassung. In diesem Artikel wird eine Methode zur Analyse und Classifizierung von Texturen, basiert "C-Calculus", vorgestellt. Die Konzepte von "C-Space" und "C-Transform" eines diskretisierten bildes werden, wenn au einfacher Art, erklärt. Eine Filtrierungsmethode, das "C-Filtering", wird präsentiert. Anwendungen dieser Methode Bilder die mehr als eine Textur enthalten, oder eine Grundtextur und einige Gegenstände, die eine eigene textur besit werden beschrieben. Schliesslich werden verschiedene experimentale Ergebnisse vorgestellt.

Résumé. Cet article présente une méthode d'analyse et de classification des textures, basée sur le "C-Calculus". N décrivons, d'une manière simple, les notions de "C-Space" et de "C-Transform" d'une image numérisée. Nous présen ensuite une méthode de filtrage appelée "C-Filtering", appliquée à des images qui contiennent plus d'une texture ou des ob texturés sur un fond texturé. Finalement nous concluons avec divers résultats expérimentaux.

Keywords. C-Calculus, C-Filter, C-Space, C-Transform.

Abstract. Based on C-Calculus, a method for analysis and classification of textures is described. Concepts of C-Space C-Transform are briefly discussed. Applications of C-Filter for filtering patterns with more than one texture, or with text objects on a textured background are described. Some direct experimental results are also given.

Introduction

In recent years numerous efforts have been made in the field of texture analysis, especially in scene descriptions and analysis of biological specimens.

The property of texture is characteristic of surfaces. This property conveys to us the information not only on the material that the surface is made of but also on its distance and position [1]. Although at present an exact scientific definition of texture is not yet found, it seems that a general concept for it is by now well established, and it is possible to describe a texture verbally, even if in approximate and empirical terms.

However if this description has to be done b computer, the problem becomes much m complex [2, 3].

An effect of texture is produced when a patt consists of a high number of subpatterns of m smaller size than that of the entire pattern. Th subpatterns must exhibit a certain density regularity, which might even be coarse over t entire visual field. They may have also their o subtextures.

At this point it is appropriate to note that t features which are extracted from the textures different methods for the purpose of text description need not necessarily refer to those subpatterns and or to their spatial distributi [4, 5].

Several authors [6–17] have proposed a classification of textures based on statistical distribution of values associated with some local properties. The discrimination of different textures would then emerge when comparing such statistical parameters. Some rather simple methods for feature extraction are listed in [1].

Although in some of these methods, standard concepts such as contrast, lines, regularity, ruggedness of the surfaces, are used, their meaning does not always coincide with that of visual perception [18].

In all these methods, noise and directions along which the different statistics are evaluated might significantly affect final results [19].

In this paper, we shall introduce an image transform and a filtering method based on C-Calculus [20] which is also used for many standard problems in pattern analysis [21, 23].

C-Calculus

The central idea of the C-Calculus has its origin in the observation that in a numerical system each member acquires a value due to its position in that system; hence each digit in such a system has two values: cardinality, and position.

If, in analogy with a string of ordinary digits, a string of sets taken out of some given system are formed, each simple set in such a string will acquire by that action a value determined by set position in the string.

We named such a string a "Composite set", or briefly, a C-set (therefore the name C-Calculus). For example:

$$a_n a_{n-1} \dots a_0$$

where $a_n, a_{n-1}, \dots, a_0$ are simple sets.

We apply the formal rules of arithmetics to operations on these strings. We select the operations of sum and product only, defined as union and intersection of sets of ordinary set calculus. By applying them to strings we obtain a commutative semi-ring, the elements of which can thus be generated in an obvious manner.

With this origin, C-Calculus is related t theory of measurement and to the physiolo certain neurons and also it seems well suited variety of other applications.

Now we will discuss some applications Calculus to pattern recognition, in particul texture analysis and to filtering of biomedica cimens [21, 22].

Suppose we have a digitized image. I represent it, in the usual way, as a square ma order n

$$A = [y_{i,j}], \quad i, j = 1, \dots, n.$$

Then a C-set for this matrix can be forme use a device called "reader" with a window This reader can read only the maximu minimum values of the function y, in the are of the matrix seen through this window. Th "w" can be thought of as a submatrix original matrix A with size w.

We scan the matrix A with the windo applying it contiguously. This scanni equivalent to the partitioning of the matrix submatrices of area "w", with an add information on the maximum and minimum of y inside of each square submatrix.

If all these submatrices are ordered in som then each one of them can be represente quadruple of values: one pair of values f position of the submatrix in the matrix, the for information on the variation of y (e values) inside of it.

An alignment of these "quadruples" string will give a C-set, for example, C_0.

This C_0 is then a representation of the image. The accuracy of representation will d on the size of the window "w": the smaller th the higher the number of squares (subma the finer the partitioning and, thus, the rep tation.

Now if such a "grid" is rigidly translated the scanner some initial phase $\delta: 0 < \delta$ second C-set, C_1 will be obtained. Then, b ing a product of C_0 and C_1, according to th of the C-Calculus, we will obtain a result wh give a finer partitioning of the original matrix and also a more accurate description of the function it contains.

An extremely valuable characteristic of C-Calculus is that it allows to reconstruct an image to the degree of precision of the instrument with which the original image has been obtained [23].

The condition for the above procedure (formation of C-sets and of their products) to lead to the convergence of the points in the image with lower values of functions to the points of local maxima in their neighbourhood is (for the unidimensional case)

$$w \leqslant \tfrac{1}{2}D + 1$$

where D is the smallest interval in the matrix in which the function "y" is monotonic. This interval is found when scanning the matrix and reading the values of maxima and minima.

C-Filter

According to condition (1), the final values associated with each point are just the highest minimum and the lowest maximum which are seen by the window when it scans the interval of width $2w - 1$ centered at that point, regardless of the scanning order. This circumstance enables us to consider a very peculiar mapping of the variation of y in the neighbourhood of some point onto the point itself. This mapping is at the base of our filtering procedure.

Suppose an image is given with one or several signals. For the signals there will be periodicities in the pattern in some regions of the image and no periodicity at all where signals are absent. The condition (1) enables us to separate these different regions very easily using different sizes of windows for scanning.

The principle which forms the basis of our filtering method can be explained as follows: for a window, the size of which is larger than the period of the signal, the scanning gives the components of the C-set with the same values of maximum and minimum of the function. Hence, as condition (1) shows, with this size of window we cannot reconstruct the periodic region of the signal. On the other hand, in the nonperiodic region the maximum and the minimum values differ. Therefore this region can be reconstructed.

Consequently, by choosing an appropriate size for the window, one can extract from our signal exclusively those components in which we are interested.

From the above, it is obvious that the dimensions of the window play a decisive role in filtering the components; namely, they ensure the convergence only in those parts of the image where the distance D in the zone of monotonicity satisfies the condition (1).

The choice of the window required for the extraction of a desired feature from an image would seem therefore to be arbitrary, or its size to be determined empirically from case to case. How to choose a proper size will be shown later.

C-Transform

In order to obtain a C-Transform of a function, the matrix containing it should be scanned.

A C-Transform gives the representation of a function or a signal in the C-Space. This space is three-dimensional with the axes:

v – for the differences of extreme values of the function inside the partitioned areas,

u – for the distance $w = \frac{1}{2}D + 1$, where D is the shortest interval in which the function is monotonic, and

t – for the frequency with which the points having equal u, v are encountered during the scanning.

C-Transform does not depend on the manner of scanning, that is, whether it is carried out row-by-row, or area-by-area.

It follows from above that a noiseless signal of constant period (axis "u") but with a varying amplitude (axis "v") will be represented by a straight line parallel to axis "u". A sine wave, i.e.

constant "u" amplitude, will have just a single point as its C-Transform in the C-Space. However, any other signal of the same period and amplitude as this sine wave will also be represented by the same point. In other words, the C-Transform of any periodic signal of constant amplitud point in the C-Space.

An image containing a signal and thu some periodicity in its pattern will have points of equal or nearly equal u, v and

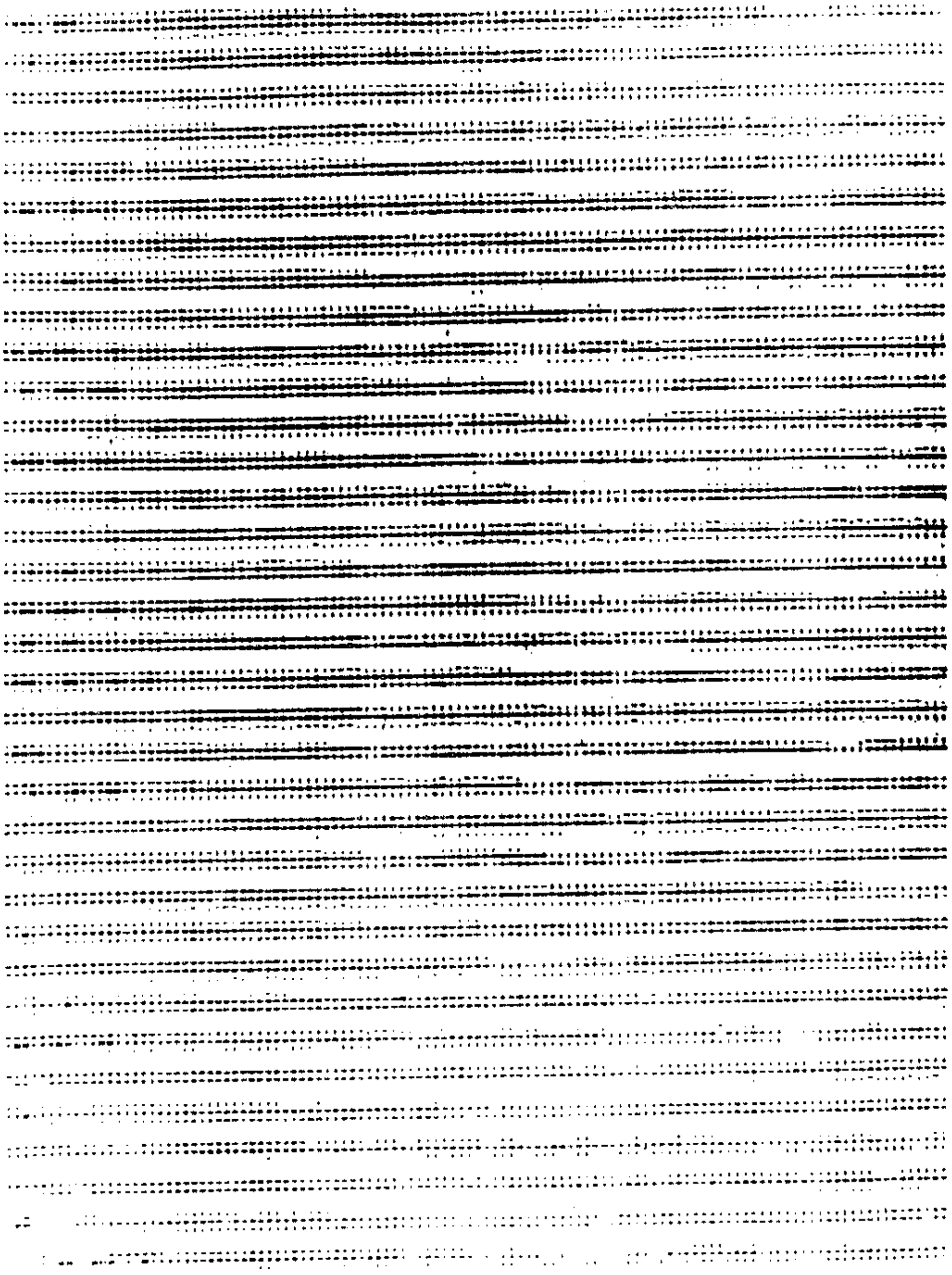

Fig. 1. Texture-input.

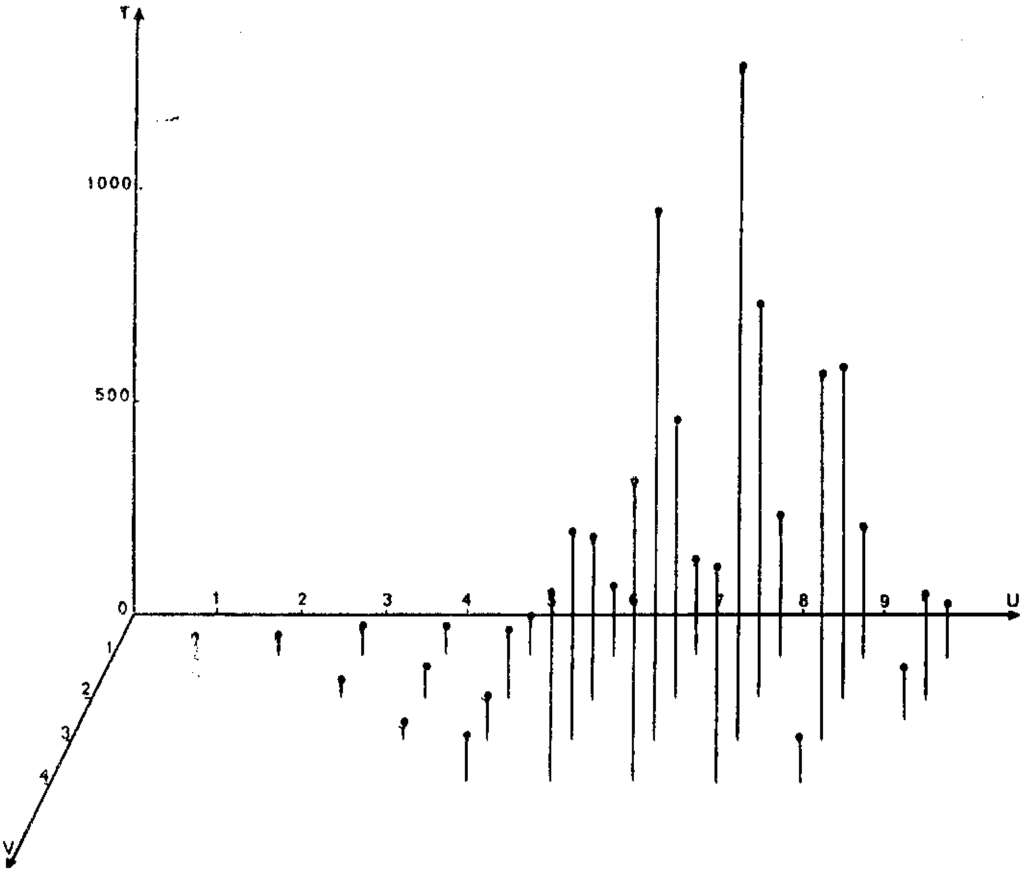

Fig. 4. C-Transform of Fig. 3.

are within the neighbourhoods "*w*" of the local maxima, and the other class with the neighbourhoods of the local maxima larger than "*w*" – is nothing else than filtering.

By choosing different sizes of "*w*" one can then think of "low-pass", "high-passs", and, even "band-pass" C-Filters, similar to the classic filters of electronics.

It appears that C-Filters possess many useful properties not all of which are yet known and which are worth investigating.

General applications

The concepts of C-Calculus seem to be extremely well suited for texture analysis. Indeed, as our experiments show, the C-Transform and the C-Filter provide us with valuable and practic tools of analysis in various applications such a

(a) Texture extraction.

(b) Extraction of objects from textur patterns.

(c) Texture filtering.

(a) *Texture extraction*

One can think of textures as models with th following characteristics.

(i) They consist of "patches" or "pieces roughly uniform either in size or in variation grey tones.

(ii) The number of such pieces is very hig compared to the number of occurrences of oth characteristics in the pattern.

In previous papers (26, . . . , 30) we discuss different types of textures to which we applied o

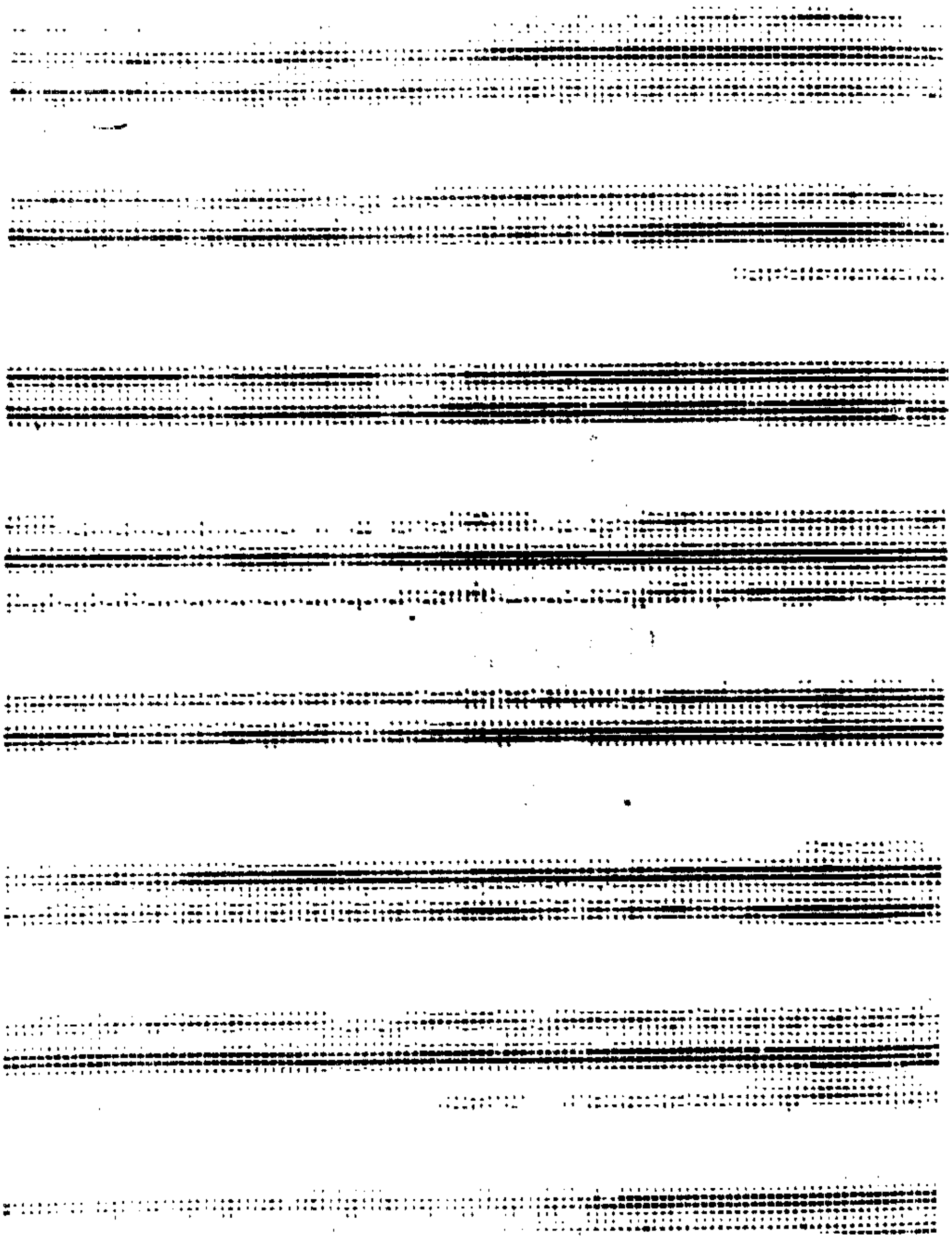

Fig. 3. Texture-input.

completely arbitrary window width produces some peculiar filtering of the image.

In other words one can state that for a certain width "w" of the scanning window, a plane $u = w$ in the C-Space divides this space into two regions: one where the function y is reconstructed, i.e. where all the points of the image contained wi the neighbourhoods "w" of local maxima ol image will lie and the other region which contain all the remaining points.

This operation of separating all points ol image into two classes: one class of points w

C-Transform will show a peak in the C-Space. Similarly any regularity in the pattern will be mapped onto more or less distinct peaks of the C-Space.

Figs. 2 and 4 show the C-Transform of the texture in Figs. 1 and 3 quantized in eight levels of grey on a 256×256 matrix.

The procedure for obtaining a C-Transform by scanning area-by-area is similar to that for scanning row-by-row.

To achieve this, the entire matrix should be scanned with a submatrix w. Then, the differences of maximum and minimum values of the function inside each submatrix should be computed and these values should be plotted on the axis v. Now on the axis "u", what should be plotted is not the distance "w" as for the scanning row-by-row, but the area w^2 of the window with which the image was scanned.

The C-Transform thus obtained does no depend on the direction in which the submatrix was shifted during the scanning. The only condition to be observed is that its successive position must be contiguous.

From the discussion of the C-Calculus, it follow that the window width plays a decisive role in th reconstruction of an image. Consider a genera case where a signal will converge only in thos regions of the picture where the distance between the grey tone extremes will satisfy th inequality (1).

Therefore an attempt to reconstruct an imag with some arbitrary window width woul immediately separate the information in th picture into two classes of regions: one where th filtering leads to convergence, and the other wher it does not. These classes can be easily distinguished so that even an attempt with

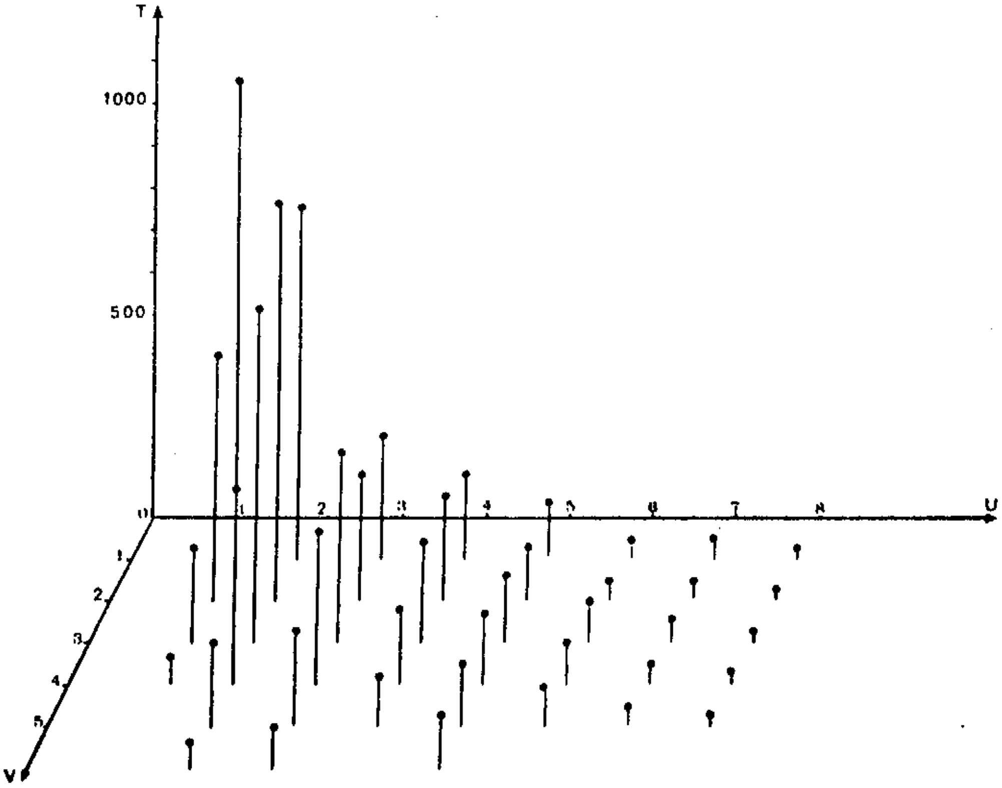

Fig. 2. C-Transform of Fig. 1.

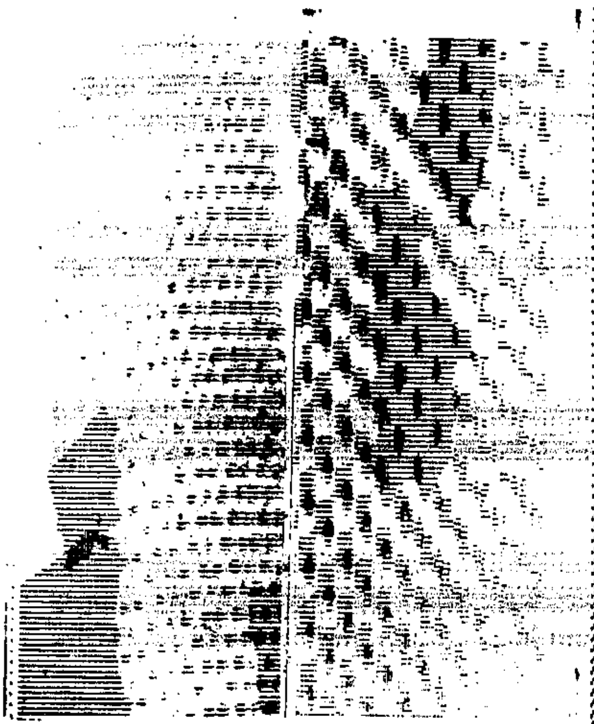

Fig. 5. Matrix-input.

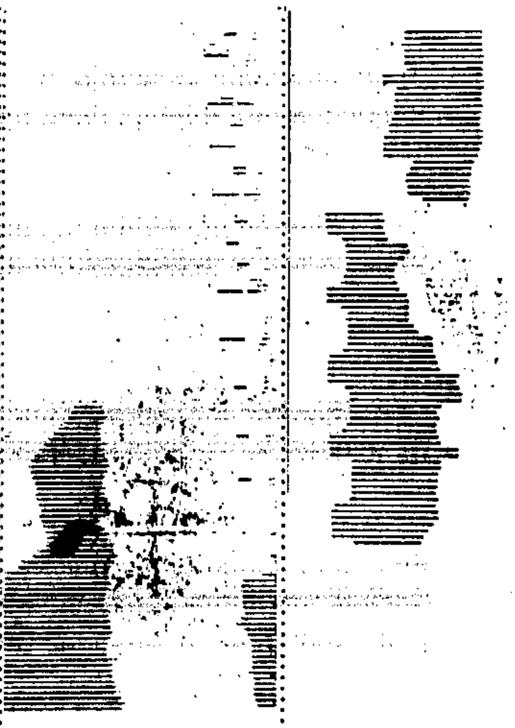

Fig. 6. Objects extract from Fig. 5.

methods of C-Transform and C-Filter. There we used eight levels of grey tone and a matrix of 256×256. C-Transforms were obtained with the well-known algorithm of clusterization [31].

As expected, C-Transforms of the textures exhibited densely clustered peaks in the C-Space, thus confirming our hypothesis on the suitability of the application of our method to textured patterns.

(b) *Extraction of objects from textured patterns*

The extraction of objects from a textured background presents another important problem in pattern analysis. To be successful in this analysis, it is imperative that the characteristics of the objects (dimensions and the variations of grey tone) should not be commensurable with those of patches composing the textured background.

In the first experiments, different types of textures were taken on which objects were superimposed. In all these cases we succeeded in extracting the objects.

The second set of experiments consisted of cases (samples) with textured background on which

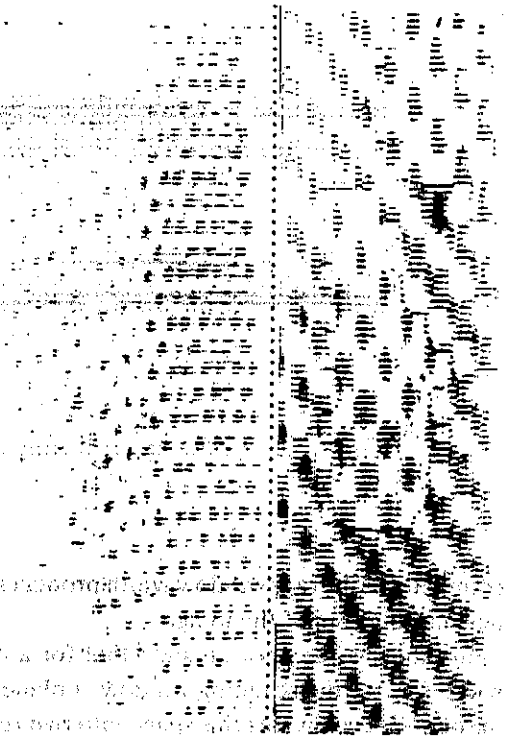

Fig. 7. Filtered part of Fig. 5.

objects with the same texture as the background were superimposed. Fig. 5 shows our input matrix 256×256 quantized to into eight levels; in Fig. 6 we have the object extracted, and Fig. 7 gives the filtered and residual information.

(c) *Texture filtering*

Numerous patterns with more than one texture were examined (see Fig. 8). A C-Transform of a pattern with two textures shows two distinct pe in the C-Space (see Fig. 9). Taking the width of scanning window $w = u$, where u is the short distance from a peak to a valley, we were abl filter out only one of the two textures. The case pattern with two textures partially superimpo was also examined.

Once more the problem of filtering one pa cular texture out of two arose. Applying the

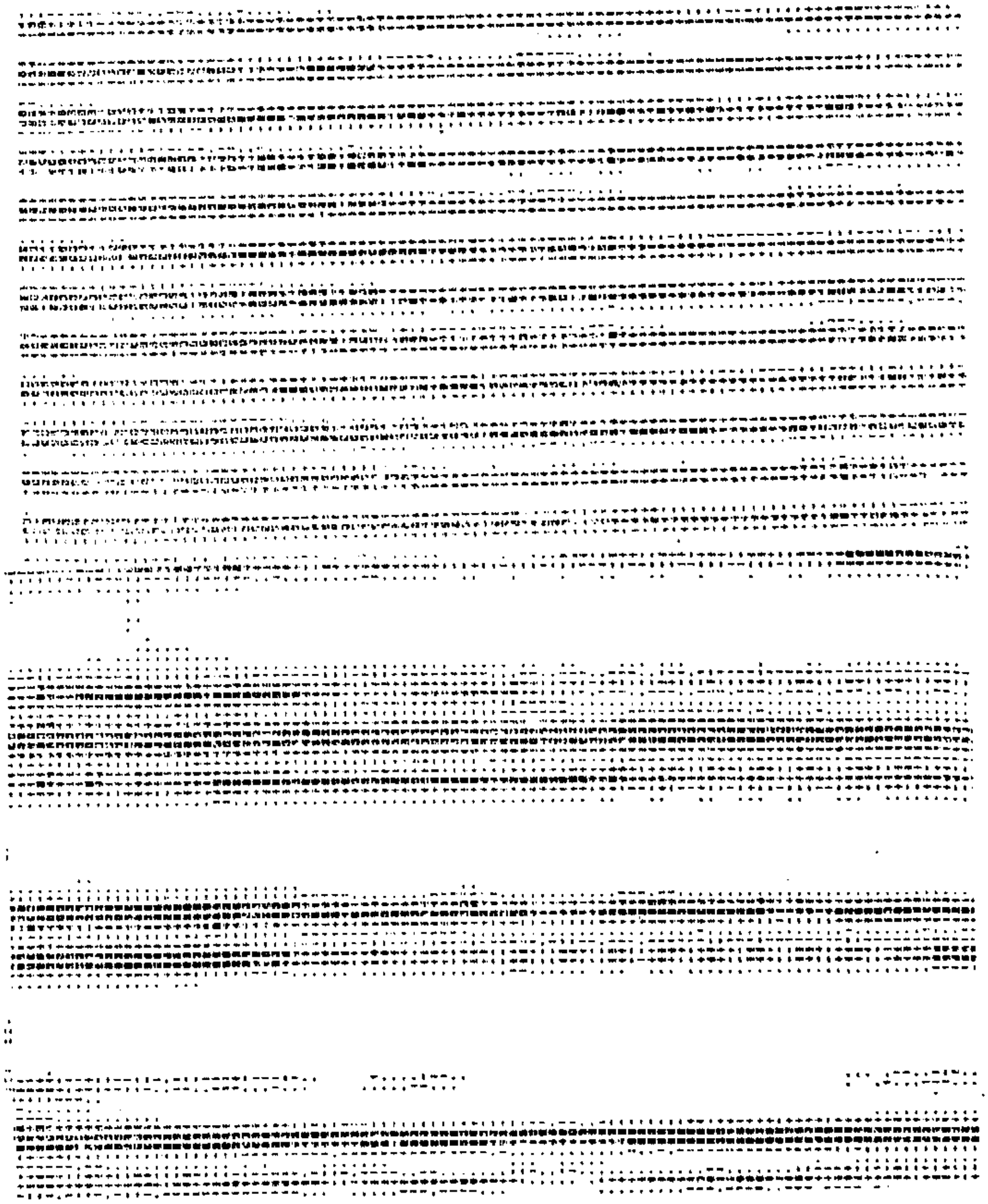

Fig. 8. Texture-input.

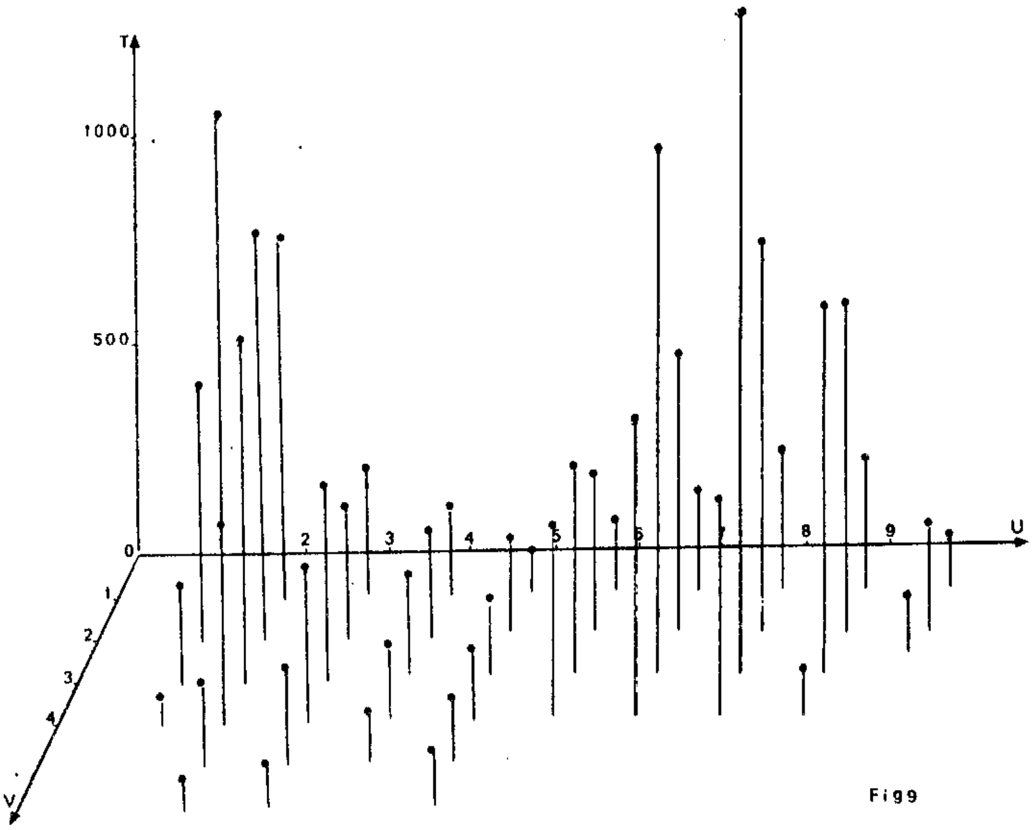

Fig. 9. C-Transform of Fig. 8.

Filter several times successively we did succeed in filtering out the texture we wanted.

Application for biomedical specimens

A scientist analysing biomedical specimens usually uses parameters of form, size, density in order to distinguish different types of cells present. The process is slow, cumbersome and not consistently reliable.

The idea of regarding a biomedical specimen as a textured pattern and of using the difference in textures for classification of cells was considered [32–34].

The encouraging results we have obtained with the C-Transform in extracting textures from multitextured patterns prompted us to apply it also to biomedical specimens and, in particular, to the problem of filtering out some "good" metaphases. The specimens we received for analysis were those of peripheral blood and of spinal cord. They w in two forms.

(a) 24×36 mm. photographic negatives,

(b) microscope slides.

To enter the slides into the computer a te camera was optically linked to a microsco Images were digitized leading to an input matri 512×512 with 16 gray levels (Fig. 10).

A C-Transform was then performed with algorithm of clusterization [31, 35]. The C-Tra form (Fig. 11) exhibited two distinct peaks in C-Space one of which was much higher than other. The high peak had low values of "v" (am litude) and of "u" (period) indicating a high le of noise in the specimen. The second peak much higher values of both u and v [36].

The latter peculiarity suggested to us to mo the procedure of our C-Filtering. Namely, w selecting the size of the window "u", we in

Fig. 10. Biological specimen input-matrix 512×512, 16 levels of grey.

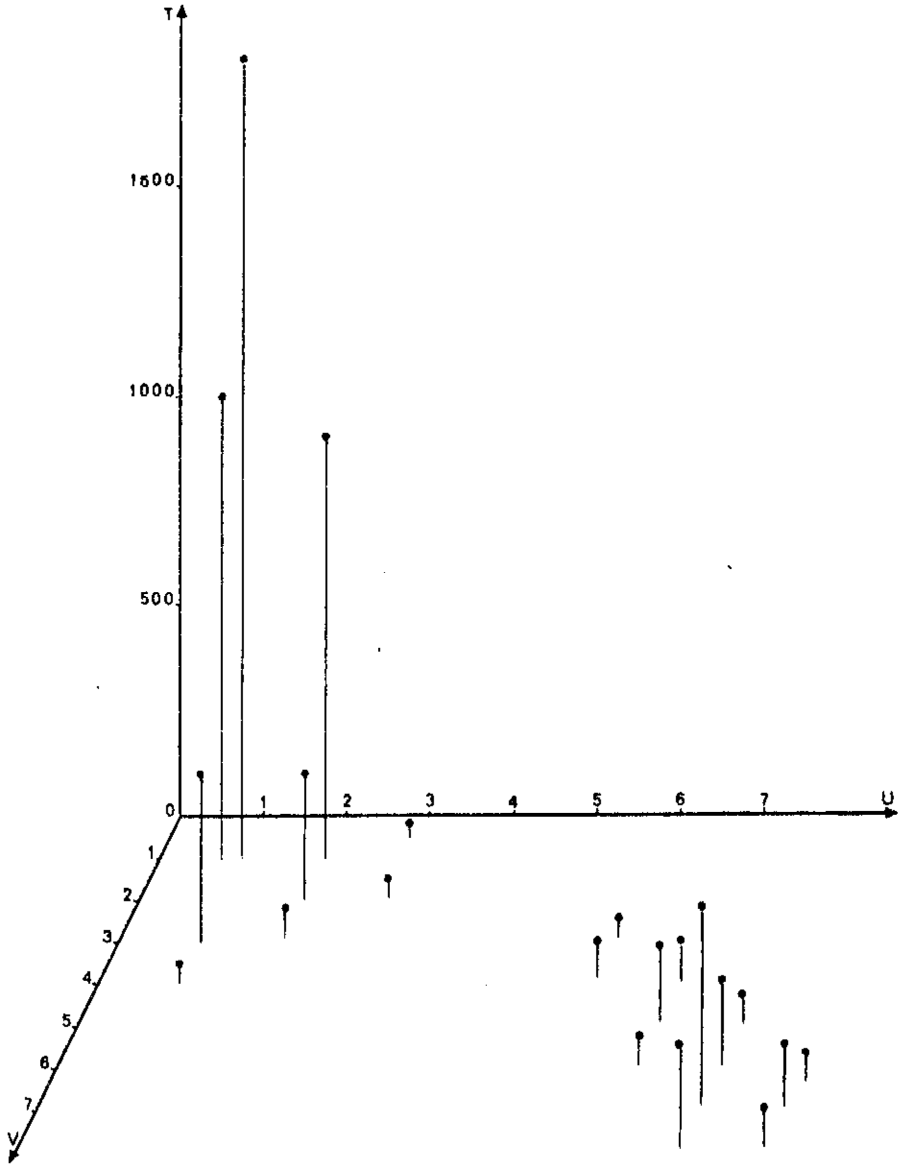

Fig. 11. C-Transform of Fig. 10.

duced a constraint "v" for the variations in the period of the signal within the window (Fig. 12).

After that, we examined 820 cells. Among them 95 were in metaphase.

After our output was filtered, we counted

(a) 9 cells not in metaphase, that is cells either retained by the filter, or so small that the periodicity of their texture was compatible with that of mitoses.

However, in each case we could se whether a cell was in a metaphase or no

(b) 97 mitoses, from which 93 w and 4 not, the error being due to t whose periodicity was very similar to mitoses.

For 95 cells in metaphase we obtained of no interest to us because they were n mitoses.

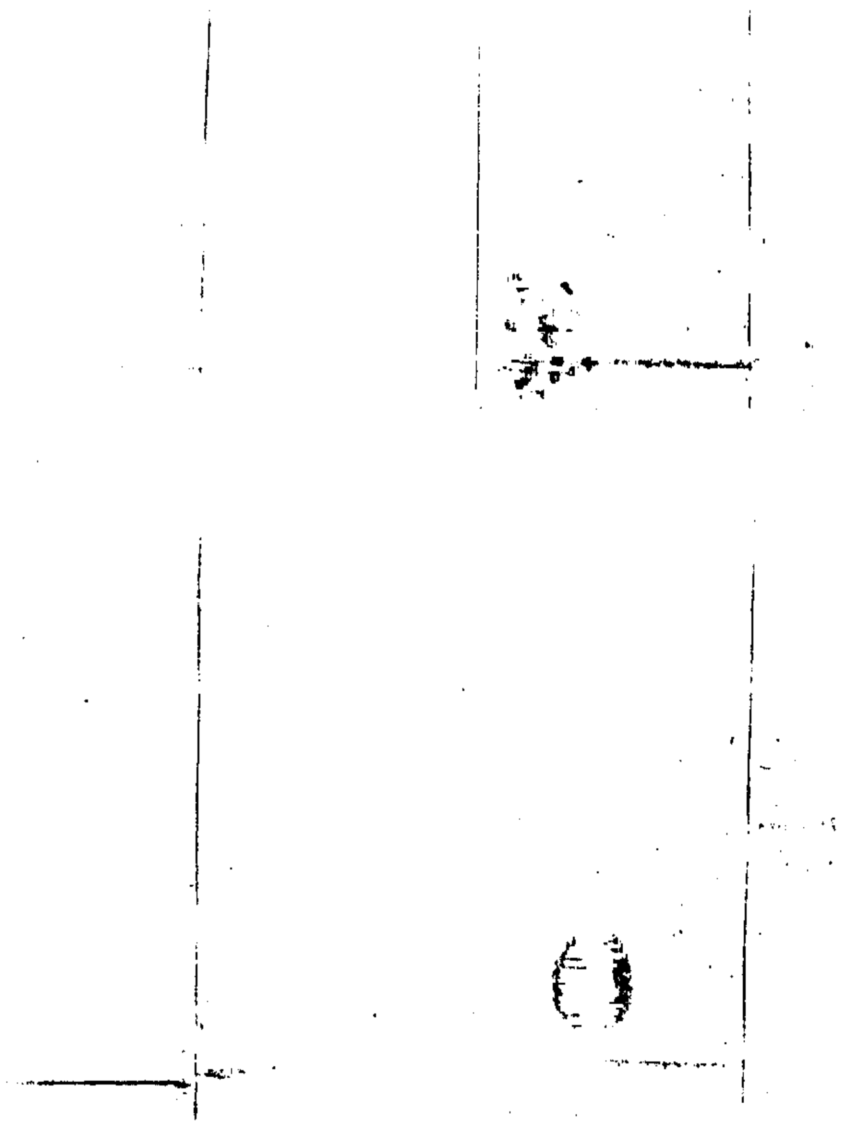

Fig. 12. Mitose extracted from Fig. 10.

Thus, in our output 1, 2% of the cells not in metaphase have been 'lost' (i.e. retained by filter), while more than 98% of such cells have been properly filtered. If now we compare the input pictures with the output pictures, the errors in the latter clearly show that the input pictures in this case were too complex as compared with the usual input.

Conclusions .

Experimental results encourage us to apply our methods not only to extraction and filtering of textures, but to modify these methods to adapt them specially to the analysis of biomedical specimens.

Our future plans hence include devising procedures especially suited for classification of chromosomes in mitoses. We intend to exploit the feature of "bandedness" which the patterns of chromosomes manifest.

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