ALGEBRAIC PATTERN RECOGNITION

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1. ALGEBRAIC STRUCTURE

It we call S a non-empty set, we can consider the $H=\{S_1, S_2, \ldots\}$ set, a set obtained by doing all the different partitions of S.
The H set is therefore clearly non-empty. In fact, it contains at least S. Let be $S_i = \{a_1, a_2, \ldots\}$ generic element of H; we can define the binary operations:

$$a_i \oplus a_j = \max(a_i, a_j) \qquad (1)$$

$$a_i \otimes a_j = \min(a_i, a_j) \qquad (2)$$

on the S_i elements. Here, max and min have the usual meaning whith regard to the cardinality of the elements.
Operations (1) and (2) possess commutative, associative and distributive properties.
It is possible to order hierarchically the elements of S_i using operations (1) and (2), and so introduce into these elements a binary relation ' $\leq$ ': reflexive, antisymmetrical and transitive. So S_i can be considered a partially ordered set whith regard to such an operation.
Now, let us suppose that the S set is discrete and finite.
Using the binary relation ' $\leq$ ', we can order the elements of S_i in pairs, so S_i is transformed into a completely ordered set, or chain.

An the $\bar{S}_i$ set can be defined the minimum and maximum elements.
The minimum element is characterised by its being less than or equal to each element of the set, and the maximum element is bigget than or equal to each element of the set.
Let us re-consider now the H set. We can now see its elements as chains. If we re-apply operations (1) and (2), we obtain an $\bar{H}$ set which satisfies the chain conditions.
The product of two of these chains does not change the order, but produce a refinement. Let us suppose, for example, that each partition contains a maximum finite number of elements.
The partition product (of these partitions) contains a greater number of elements with regard to the partition factor.
Remembering that a partially ordered set is called a lattice if the r-intersection and r-union of each pair of its elements exists, it can be said that $\bar{H}$ with the operations introduced into it is a lattice.

2. CONVERGENCE AND FILTERING

The transmission and elaboration of signals, of any dimension, has led to the proliferation of many algorythms more or less 'ad hoc'.
We have proposed the use of the algebraic structure, (see paragraph 1) in the successive phases which characterize the elaboration of signals.
An example, although simple, allows us to illustrate the idea which has inspired our research to the reader: let us consider a totally ordered, one-dimensional vectorial space V_1 (fig.1), and on it we produce a partition S_1, which satisfies both the chain conditions and the lattice.
If S_2 is a second distinct partition of S_1, the Cartesian product of these two chains produces a partition-chain, which is a refinement of the partitions-chains-factors.
On fig.2 we demostrate the Cartesian product of S_1 and S_2.

fig.1: V_1 a totally ordered vectorial space V_1 |––––––––––|
S_1 a chain made up of the elements of V_1 S_1 . . . .
S_2 a chain made up of the elements of V_1 S_2 * * * * * * *

Fig.2
S_1 | | | | | | | | |
S_2 * * * * * * * *
$S_1 \wedge S_2$ $ $$ $ $ $$ $ $ $ $

By repeating the procedure we obtain the required refinement. If the vectorial space is discrete the procedure finishes after a finite number of products, and so we obtain all the points in which V_1 has been sampled. It is also clear that the number of products to be done is limited to the dimensions of the elements of the partition.
Let us consider a function $u(x)$, (fig.3), monotone.
Here, we can produce a partition of both the axis x and the axis $y=u(x)$.
We can clearly state that the partitions obtained in this way satisfy the chain conditions.

fig.3

Remembering that in a chain the minimal and the maximal coincide with the extremes of the element of partition, we can now represent the function u(x) as a string S_1 of quadruples:

$$u(x) \equiv (x_1,x_2,y_1,y_2)\ldots(x_i,x_{i+1},y_j,y_{j+1})$$

where x_i and x_{i+1} are respectively the minimal and the maximal of the i-th element obtained by partitioning the axis of x, and where $y_j=u(x)$ and $y_{j+1}=u(x_{i+1})$ are respectively the minimal and the maximal of the i-th element obtained by partitioning the axis of y.

Let us consider a second string S_2, different from the first one. If we realize the cartesian product $S_1 \wedge S_2 \equiv P_1$

we can affirm that P_1 (see the cartesian chain product) represents a refinement not only of the axis of x but also of the axis of y, in short of the function u(x).

By repeating the procedure we obtain the desired refinement of u(x).

If the function u(x) is discrete the maximum obtainable refinement coincides with the sampling used to digitise the u(x).

Remembering that the properties which the Cartesian chain product has do not depend on the dimensions of the space in which the elements belonging to the chain itself are represented, we can also affirm that it is also true for this kind of function $u(x_1,x_2,\ldots,x_n)$ therefore, it is also valid for the pictures $u(x_1,x_2)$.

Let us consider, for simplicity sake a generic function u(x) whose domain is one-dimensional. This question springs to mind: it is still possible to obtain a refinement of the function u(x) which is as precise as we would like it to be.

This could be the answer: we can produce a partition of the domain of the function u(x) so that in each element of the partition the function is monotone or, in other words, we impose the condition that the dimension of each element of the partition, which from now on will all be considered equal in the same partition, must be half of the smallest zone of monotonicity contained in the signal. We can therefore construct a certain number of partitions. This number dipends an the dimensions of the smallest zones of monotonicity in the signal, which satisfy the chain conditions.

The Cartesian product of these chains ensures us both the convergence of the procedure and the maximum possible refinement of the signal.

For those interested in this argument, see (1) for a different demonstration of this idea.

From now on, we will study only the discrete functions and therefore the maximum possible refinement will be the sampling one.

The lattice theory allows us to state that the above mentioned considerations do not depend on the dimensions of the domain.

Since in this article we intend to concentrate our attention on Pattern Recognition, in the following paragraphs, we will look at signals whose domain is one or two-dimensional.

We have a signal and one of its partitions whose elements all have the same dimension (see fig.4).

We ask ourselves if it is possible to reconstruct the entire signal, as precisely as we would like.

An answer springs immediately to mind, as we have shown in the preceding paragraphs we can say that only the zones in which the condition is satisfied will be reconstructed.

In this way we have introduced the idea which has guided us in the realisation of a digital filter whose tranfer function can be thus described: the filter gives us all the zones of the signal (as precisely as we would like) in which the condition is satisfied (2), and removes all the after zones in which the condition is not satisfied. (fig.5).

3. MATRIX OF THE FEATURES

Now, we have to solue the problem of how to determine the dimensions of the element of partition, so that we can filter or re-construct the signal.From now on, we will adopt the work 'window' in place of the element of a partition.

Let us consider a signal u(x), discrete and whose domain D, is finite and one-dimensional,and realize a partition of the signal with a window which contains only two points, that is:

$$w = x_i x_{i+j} \quad \forall x_i \in D \quad \text{and} \quad j=1$$

Definition. Let us define the following measures carried out on the signal 'dynamic':

$$d = |u(x_i) - u(x_{i+j})| \quad \forall x_i \in D$$

We hope that the reader has recognised in the definition, the operator Δ 'differential' introduced into 'finite difference equations'(3).

Now, let us introduce the vector n-dimensional, whose first component is relative to the value $d=0$, the second one to the value $d=1$,etc.

The cardinality of the component is the frequence with which the dynamic d=0,...,n occours with relation to the partition under investigation,w=2.
If we condider a second window:
$w=x_i x_{i+j}$ $\forall x_i \in D$ and j=2, we obtain a second vector.
The vector obtained for the different values of $j \in A$, with A finite because the signal is discrete and finite, constitute the rows of our matrix, the columns of which -also finite- have as indexes the different values of the dynamic.
Definition. Let us define the 'shape' of the signal:

$$\text{shape} = \frac{|u(x_{i+j})-u(x_i)|}{x_{i+1} - x_i} \quad \text{(fig.6.7.8.9)}$$

4. SOME REMARKS

In this paragraph we wish to present the experimental results obtained by the application of the method proposed in the preceding paragraphs on some of the most meaningfulphases which characterise the entire process of pattern recognition.

4.1.Contours

The perhaps too copious literature with regard to Pattern Recognition proposes several definitions of the contour of an object and at least as many algorythms which are more are less suitable for the extraction of it.
We agree in part with Pavlidis (4) when he in, reference to the theory of Gestalt, sustains the difficulty in the perception of the contour and a rigorous definition of it. We agree with him a little less when he sustains that the use of local operators, such as the gradient or the laplacian, permits the determination of it.
Amongst the varous approaches, some openly 'ad hoc', the 'facet model', even if it is time consuming, seems to us to be the most worthy of our attention.
Let us begin with a definition of a region of the visual field.
Definition. A part of the visual field is defined as a region if it satisfies a predicate of uniformity (e.g. the same tone of grey,texture, contour, etc.).
This is our definition of contour.
Definition. The 'region' existing between two distinct contigous regions of the visual field is defined as a 'contour'.
Since it is possible to associate a function with each region, the level of the corresponding polynomial of approximation is linked to the major or minor complexity of the region, it can be deduced that there is a noticeable 'variation' of such a function in the contour zone.

The 'variation' which we propose in our algorythm for the extraction of the contour is the 'shape' of the function.
Is it, in fact, from the matrix of the features that we can determine the shape of the function, and by the application of our filter to the whole picture we can re-construct only the contour zone.
We believe that a further observation is opportune at this stage: after having determined in the matrix of the features the dimension of the window suitable for filtering, we choose the dynamic inside the window so that it as greater than or equal to a dynamic-threshold.
The choise of this dynamic-threshold is made by returning once again to the matrix of the features. In fact, the dimensions of the window individuate, in an unambiguous very, a row of our matrix and from this we can choise the most suitable dynamic.
A rigorous formalisation of the matrix of the features and of its inverse (from the matrix to obtain the reconstruction, point by point, of the signal) is the theme of a paper which will appear shortly.

4.2.Textures

The study and further classification of textures can be considered a science 'in itself'. In fact, texture is of not little importance in psychophysiological studies of visual perception.(5)
Pattern Recognition contains several algorythms which are proposed by the study of textures.
A classification, even if it is imprecise, of these algorythms, could be the following:

a) linguistic approach (6)
b) statistical aprroach (7)
c) structural approach (7)

A precise definition of texture is not easy to give using layman's language, and it becomes extremely difficult when using mathematical language. Models suitable to describe textures have been proposed by several researches.
Amongst these there is the model proposed by Prof.R.M.Haralick (8), who suggests that there is a relationship between the texture and movement of a particle- relation of De Broglie-.
This models seems to us to be one of the most interesting, and offers us some sort of relatioship with the algorythm proposed by us.
In one of our previous works, we listed the conditions that a region must satisfy to be considered a texture:

a) the texture must contain 'pieces' which have more or less the same 'shape';
b) then must be a spatial disposition law of the 'pieces' in the entire region;

c) the number of 'pieces' contained in the region must be meaningful. We would like, before illustrating the essential phases of our algorytm, to show the matrix of the features relative to a few artificial textures. (Fig.10.11.12)
We would now like to describe the essential steps of our algorythm of the classification of textures:

i) determination, from the matrix of the features, of the shape of the piece;

ii)determination of the distance between the pieces in the visual field.

We believe that some experimental results obtained with natural textures are particulary suitable for a more complete understanding of our algorythm.
A confrontation between the algorythm proposed by Prof.Haralick and ours is at an advanced stage, using as pictures those proposed by Brodatz in his book (9.).

4.3.Extraction of the features

The extraction of the features and their further classification are obligatory steps in the recognition process of models.
In this phase, the main point to be resolved is the choice between various fragments from a model: the problem is to choose those which contain the greatest amount of information. It must be pointed out that the valuation of the amount of information constitutes a problem which is not easily resolvable, given the absolute lack of references, since this valuation is totally subjective.
The algorythm proposed by us can be set out in this way:

a) extraction of the fragments relative to the contour zones in which there is a marked curvature;

b) elimination of those fragments which do not satisfy a particular condition-threshold on contrast.

We believe that a 'local' measure of the curvature must be linked to the global behaviour of the entire contour.
As for the contrast, we have already been able to see, by presenting the algorythm for the extraction of the contour, now it plays as important role in the description of the features of a picture. A further selection enables us to improve the 'meaningfulness' of the extracted fragments. There are different measures of con trast in use. Later, we will refer to the one used in neurophysiology (10):

$$C = \frac{\Delta B}{B} : \frac{B_e - B}{B}$$

where B_e and B indicate respectively the luminance of the object and of the region immediately surrounding it.
We believe that some indications as to the determination of the 'window' suitable for the realisation of the above mentioned steps are useful. The chosen window are indicated respectively by w_1 and w_2, for the extraction of the contour and for the test on contrast where:

$$w_1 = \begin{cases} w^* \text{ if } w^* \text{ is odd} \\ w^* + 1 \quad \text{otherwise} \end{cases}$$

w_2 in such a way that it tends to one relationship

$$\frac{w_2^2 - w_1^2}{w_1^2}$$

Where w^* is the window chosen from the matrix of the features for the extraction of the contour. The selection of the points of curvature is obtained by scanning with window w, each point, P_c, belonging to the contour of the object. The scansion window is centred on P_c over point P_1 and P_2, (the existence of these pairs of points is ensured by the observation that the contour of an object is a closed line) which intersect the lateral edge of the window.
The condition:

$$d_4(P_1,P_2) = |i_1 - i_2| + |j_1 - j_2| < w_1 \qquad \begin{matrix} i_1 \neq i_2 \neq i \\ j_1 \neq j_2 \neq j \end{matrix}$$

where i,j indicate respectively the row index and column index of P_c, we are ensured the selection of the points of curvature of the contour. On each point chosen in the previous step a test is done on the contrast in this way:
P_c is a point of the contour there it, the two windows w_1 and w_2 are centered. The dynamic existing whitin the areas $w_2 - w_1$ and w_1 are indicated respectively by d_e and d_i, the condition that point P_c must satisfy so that it is chosen as the centre of an informative fragment is the following: $|d_e - d_i| \geq d^*$
where d^* corrisponds to the chosen dynamic of the matrix of the features for the extraction of the contour.

5. CONCLUSIONS

We have alrady underlined several times in this particular work how the universe of the recognition of model is starred by an eccess of algorythms more or less suitable for the resolution of a very small part of the whole process, and very aften only of a rather narrow field of models.
The absence of a solid theory which not only justifies the various sorts of existing algorythm but which enables a confrontation between them is enaugh to justify the doubts and criti-

cisms about the 'science of the recognition of models'.
The existence of a unique theory which justifies and which permits a formalisation of the various phases presented in this work is of small confort to us.
We would now like to make a list of a few applications of this theory and various real problems:
1)analysis of the rugosity of materials (11)
2)analysis of the cut surface obtained with a laser
3)analysis of palatine marks (12)
4)biomedical applications (13): a)classification of human banded chromosomes; b) elaboration of pictures from T.A.C.; c) elaboration of ecographic pictures; d) elaboration of thermographic pictures.

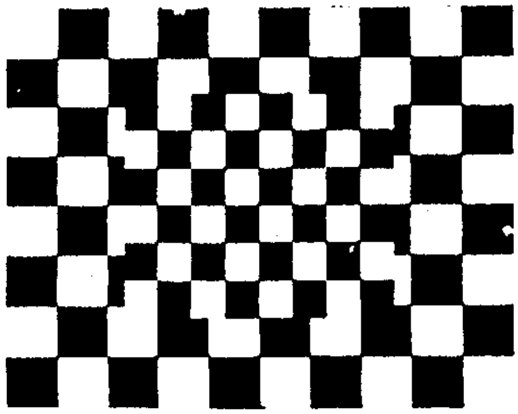


Fig.4 Input signal Fig.5 Filtered signal

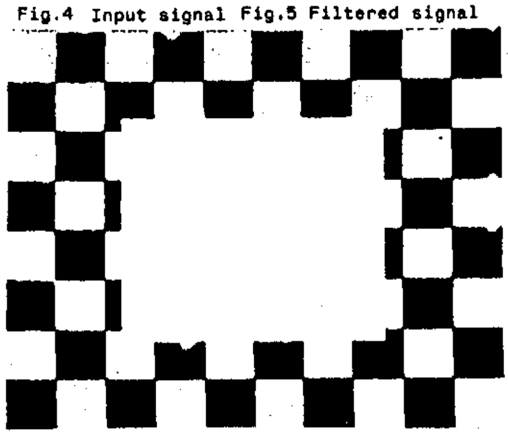
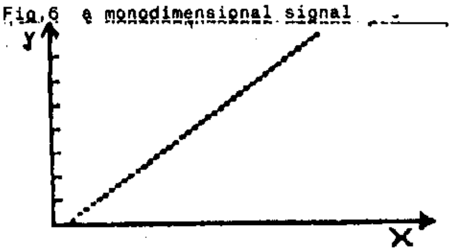


Fig.6 a monodimensional signal



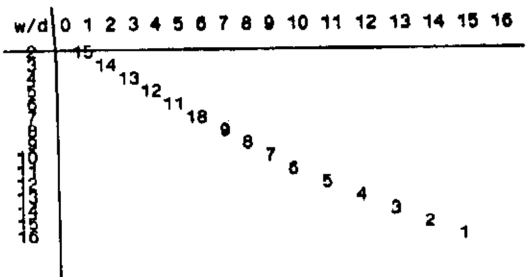


Fig.7 its matri of features

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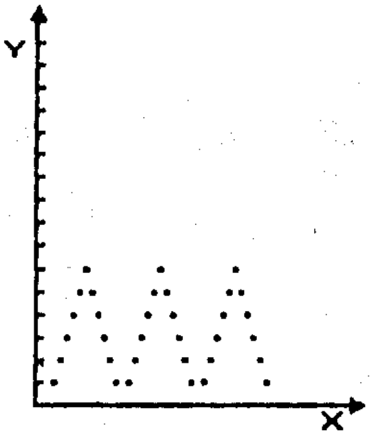

Fig.8 a monodimensional signal

w/d	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2			35														
3			5		29												
4					10		23										
5					5		10		17								
6							10		10		11						
7							5		10		10		5				
8									10		10		9				
9									5		10		13				
10											10		17				
11											5		21				
12													25				
13													24				
14													23				
15													22				
16													21				

Fig.9 its matrix of features

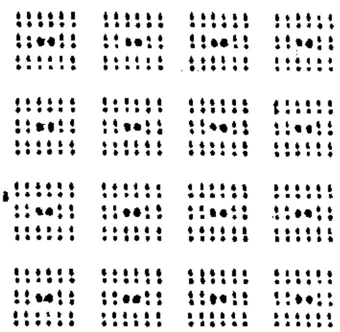

Fig.10

a two dimensional signal and its matrix of features

w/d	0	1	2	3	4	5	6	7	8	9	10	11
2						769						
3						325					351	
4											625	
5											570	
6											529	
7											484	
8											441	
9											482	
10											301	

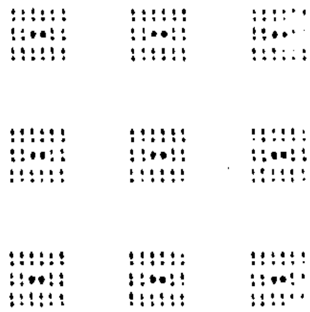

Fig.11

a two dimensinal signal and its matrix

w/d	0	1	2	3	4	5	6	7	8	9	10	11
2	483					361						
3	255					350					171	
4						405					324	
5						192					484	
6											625	
7											576	
8											529	
9											484	
10											441	

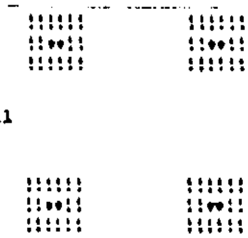

Fig.12

a two dimensional signal and its matrix

w/d	0	1	2	3	4	5	6	7	8	9	10	11
2	736					225						
3	576					219					105	
4	400					245					185	
5	200					287					289	
6						329					400	
7						147					529	
8											625	
9											576	
10											529	

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on the S_i elements. Here, max and min have the usual meaning whith regard to the cardinality of the elements.

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The partition product (of these partitions) contains a greater number of elements with regard to the partition factor.

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On fig.2 we demostrate the Cartesian product of S_1 and S_2.

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S_1 a chain made up of the elements of V_1
S_2 a chain made uo of the elements of V_1

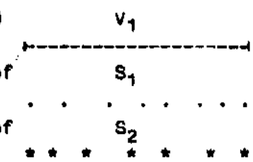
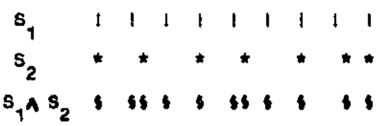


Fig.2



By repeating the procedure we obtain the required refinement. If the vectorial space is discrete the procedure finishes after a finite number of products, and so we obtain all the points in which V_1 has been sampled. It is also clear that the number of products to be done is limited to the dimensions of the elements of the partition.

Let us consider a function u(x), (fig.3),monotone. Here, we can produce a partition of both the axis x and the axis y=u(x).

We can clearly state that the partitions obtained in this way satisfy the chain conditions.

fig.3

Remembering that in a chain the minimal and the maximal coincide with the extremes of the element of partition, we can now represent the function u(x) as a string S_1 of quadruples:

$$u(x) \equiv (x_1,x_2,y_1,y_2)\ldots(x_i,x_{i+1},y_j,y_{j+1})$$

where x_i and x_{i+1} are respectively the minimal and the maximal of the i-th element obtained by partitioning the axis of x, and where $y_j=u(x)$ and $y_{j+1}=u(x_{i+1})$ are respectively the minimal and the maximal of the i-th element obtained by partitioning the axis of y.

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Remembering that the properties which the Cartesian chain product has do not depend on the dimensions of the space in which the elements belonging to the chain itself are represented, we can also affirm that it is also true for this kind of function $u(x_1,x_2,\ldots,x_n)$ therefore, it is also valid for the pictures $u(x_1,x_2)$.

Let us consider, for simplicity sake a generic function u(x) whose domain is one-dimensional. This question springs to mind: it is still possible to obtain a refinement of the function u(x) which is as precise as we would like it to be.

This could be the answer: we can produce a partition of the domain of the function u(x) so that in each element of the partition the function is monotone or, in other words, we impose the condition that the dimension of each element of the partition, which from now on will all be considered equal in the same partition, must be half of the smallest zone of monotonicity contained in the signal. We can therefore construct a certain number of partitions. This number depends an the dimensions of the smallest zones of monotonicity in the signal, which satisfy the chain conditions.

The Cartesian product of these chains ensures us both the convergence of the procedure and the maximum possible refinement of the signal.

For those interested in this argument, see (1) for a different demonstration of this idea.

From now on, we will study only the discrete functions and therefore the maximum possible refinement will be the sampling one.

The lattice theory allows us to state that the above mentioned considerations do not depend on the dimensions of the domain.

Since in this article we intend to concentrate our attention on Pattern Recognition, in the following paragraphs, we will look at signals whose domain is one or two-dimensional.

We have a signal and one of its partitions whose elements all have the same dimension (see fig.4). We ask ourselves if it is possible to reconstruct the entire signal, as precisely as we would like.

An answer springs immediately to mind, as we have shown in the preceding paragraphs we can say that, only the zones in which the condition is satisfied will be reconstructed.

In this way we have introduced the idea which has guided us in the realisation of a digital filter whose tranfer function can be thus described: the filter gives us all the zones of the signal (as precisely as we would like) in which the condition is satisfied (2), and removes all the after zones in which the condition is not satisfied. (fig.5).

3. MATRIX OF THE FEATURES

Now, we have to solue the problem of how to determine the dimensions of the element of partition, so that we can filter or re-construct the signal. From now on, we will adopt the work 'window' in place of the element of a partition.

Let us consider a signal u(x), discrete and whose domain D, is finite and one-dimensional, and realize a partition of the signal with a window which contains only two points, that is:

$$w = x_i x_{i+j} \quad \forall x_i \in D \quad \text{and} \quad j=1$$

Definition. Let us define the following measures carried out on the signal 'dynamic':

$$d = \left| u(x_i) - u(x_{i+j}) \right| \quad \forall x_i \in D$$

We hope that the reader has recognised in the definition, the operator Δ 'differential' introduced into 'finite difference equations' (3).

Now, let us introduce the vector n-dimensional, whose first component is relative to the value $d=0$, the second one to the value $d=1$, etc.

The cardinality of the component is the frequence with which the dynamic $d=0,\dots,n$ occoure with relation to the partition under investigation, $w=2$.
If we condider a second window:
$w=x_i x_{i+j} \ \forall x_i \in D$ and $j=2$, we obtain a second vector.
The vector obtained for the different values of $j \in A$, with A finite because the signal is discrete and finite, constitute the rows of our matrix, the columns of which -also finite- have as indexes the different values of the dynamic.
Definition. Let us define the 'shape' of the signal:

$$\text{shape} = \frac{\left| u(x_{i+j}) - u(x_i) \right|}{x_{i+1} - x_i} \quad \text{(fig.6.7.8.9)}$$

4. SOME REMARKS

In this paragraph we wish to present the experimental results obtained by the application of the method proposed in the preceding paragraphs on some of the most meaningfulphases which characterise the entire process of pattern recognition.

4.1.Contours

The perhaps too copious literature with regard to Pattern Recognition proposes several definitions of the contour of an object and at least as many algorythms which are more are less suitable for the extraction of it.
We agree in part with Pavlidis (4) when he in, reference to the theory of Gestalt, sustains the difficulty in the perception of the contour and a rigorous definition of it. We agree with him a little less when he sustains that the use of local operators, such as the gradient or the laplacian, permits the determination of it.
Amongst the varous approaches, some openly 'ad hoc', the 'facet model', even if it is time consuming, seems to us to be the most worthy of our attention.
Let us begin with a definition of a region of the visual field.
Definition. A part of the visual field is defined as a region if it satisfies a predicate of uniformity (e.g. the same tone of grey,texture, contour, etc.).
This is our definition of contour.
Definition. The 'region' existing between two distinct contigous regions of the visual field is defined as a 'contour'.
Since it is possible to associate a function with each region, the level of the corresponding polynomial of approximation is linked to the major or minor complexity of the region, it can be deduced that there is a noticeable 'variation' of such a function in the contour zone.

The 'variation' which we propose in our algorythm for the extraction of the contour is the 'shape' of the function.
Is it, in fact, from the matrix of the features that we can determine the shape of the function, and by the application of our filter to the whole picture we can re-construct only the contour zone.
We believe that a further observation is opportune at this stage: after having determined in the matrix of the features the dimension of the window suitable for filtering, we choose the dynamic inside the window so that it as greater than or equal to a dynamic-threshold.
The choise of this dynamic-threshold is made by returning once again to the matrix of the features. In fact, the dimensions of the window individuate, in an unambiguous very, a row of our matrix and from this we can choise the most suitable dynamic.
A rigorous formalisation of the matrix of the features and of its inverse (from the matrix to obtain the reconstruction, point by point, of the signal) is the theme of a paper which will appear shortly.

4.2.Textures

The study and further classification of textures can be considered a science 'in itself'. In fact, texture is of not little importance in psychophysiological studies of visual perception.(5) Pattern Recognition contains several algorythms which are proposed by the study of textures.
A classification, even if it is imprecise, of these algorythms, could be the following:

a) linguistic approach (6)
b) statistical aprroach (7)
c) structural approach (7)

A precise definition of texture is not easy to give using layman's language, and it becomes extremely difficult when using mathematical language. Models suitable to describe textures have been proposed by several researches.
Amongst these there is the model proposed by Prof.R.M.Haralick (8), who suggests that there is a relationship between the texture and movement of a particle- relation of De Broglie-. This models seems to us to be one of the most interesting, and offers us some sort of relationship with the algorythm proposed by us.
In one of our previous works, we listed the conditions that a region must satisfy to be considered a texture:

a) the texture must contain 'pieces' which have more or less the same 'shape';
b) then must be a spatial disposition law of the 'pieces' in the entire region;

c) the number of 'pieces' contained in the region must be meaningful. We would like, before illustrating the essential phases of our algorytm, to show the matrix of the features relative to a few artificial textures. (Fig.10.11.12) We would now like to describe the essential steps of our algorythm of the classification of textures:

i) determination, from the matrix of the features, of the shape of the piece;

ii)determination of the distance between the pieces in the visual field.

We believe that some experimental results obtained with natural textures are particulary suitable for a more complete understanding of our algorythm.

A confrontation between the algorythm proposed by Prof.Haralick and ours is at an advanced stage, using as pictures those proposed by Brodatz in his book (9.).

4.3.Extraction of the features

The extraction of the features and their further classification are obligatory steps in the recognition process of models.

In this phase, the main point to be resolved is the choice between various fragments from a model: the problem is to choose those which contain the greatest amount of information. It must be pointed out that the valuation of the amount of information constitutes a problem which is not easily resolvable, given the absolute lack of references, since this valuation is totally subjective.

The algorythm proposed by us can be set out in this way:

a) extraction of the fragments relative to the contour zones in which there is a marked curvature;

b) elimination of those fragments which do not satisfy a particular condition-threshold on contrast.

We believe that a 'local' measure of the curvature must be linked to the global behaviour of the entire contour.

As for the contrast, we have already been able to see, by presenting the algorythm for the extraction of the contour, now it plays as important role in the description of the features of a picture. A further selection enables us to improve the 'meaningfulness' of the extracted fragments. There are different measures of con trast in use. Later, we will refer to the one used in neurophysiology ('10):

$$C = \frac{\Delta B}{B} : \frac{B_e - B}{B}$$

where B_e and B indicate respectively the luminance of the object and of the region immediately surrounding it.

We believe that some indications as to the determination of the 'window' suitable for the realisation of the above mentioned steps are useful. The chosen window are indicated respectively by w_1 and w_2, for the extraction of the contour and for the test on contrast where:

$$w_1 = \begin{cases} w^* & \text{if } w^* \text{ is odd} \\ w^*+1 & \text{otherwise} \end{cases}$$

w_2 in such a way that it tends to one relationship

$$\frac{w_2^2 - w_1^2}{w_1^2}$$

Where w^* is the window chosen from the matrix of the features for the extraction of the contour. The selection of the points of curvature is obtained by scanning with window w, each point, P_c, belonging to the contour of the object. The scansion window is centred on P_c over point P_1 and P_2, (the existence of these pairs of points is ensured by the observation that the contour of an object is a closed line) which intersect the lateral edge of the window.

The condition:

$$d_4(P_1,P_2) = |i_1 - i_2| + |j_1 - j_2| < w_1 \quad \begin{matrix} i_1 \neq i_2 \neq i \\ j_1 \neq j_2 \neq j \end{matrix}$$

where i,j indicate respectively the row index and column index of P_c, we are ensured the selection of the points of curvature of the contour. On each point chosen in the previous step a test is done on the contrast in this way: P_c is a point of the contour there it, the two windows w_1 and w_2 are centered. The dynamic existing whitin the areas $w_2 - w_1$ and w_1 are indicated respectively by d_e and d_i, the condition that point P_c must satisfy so that it is chosen as the centre of an informative fragment is the following: $|d_e - d_i| \geq d^*$

where d^* corrisponds to the chosen dynamic of the matrix of the features for the extraction of the contour.

5. CONCLUSIONS

We have alrady underlined several times in this particular work how the universe of the recognition of model is starred by an eccess of algorythms more or less suitable for the resolution of a very small part of the whole process, and very aften only of a rather narrow field of models.

The absence of a solid theory which not only justifies the various sorts of existing algorythm but which enables a confrontation between them is enaugh to justify the doubts and criti-

cisms about the 'science of the recognition of models'.
The existence of a unique theory which justifies and which permits a formalisation of the various phases presented in this work is of small confort to us.
We would now like to make a list of a few applications of this theory and various real problems:
1)analysis of the rugosity of materials (11)
2)analysis of the cut surface obtained with a laser
3)analysis of palatine marks (12)
4)biomedical applications (13): a)classification of human banded chromosomes; b) elaboration of pictures from T.A.C.; c) elaboration of ecographic pictures; d) elaboration of thermographic pictures.

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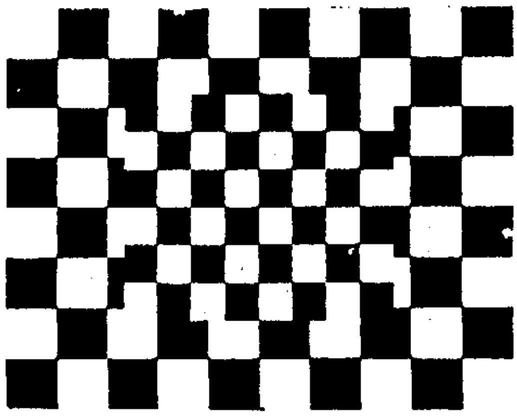


Fig.4 Input signal Fig.5 Filtered signal

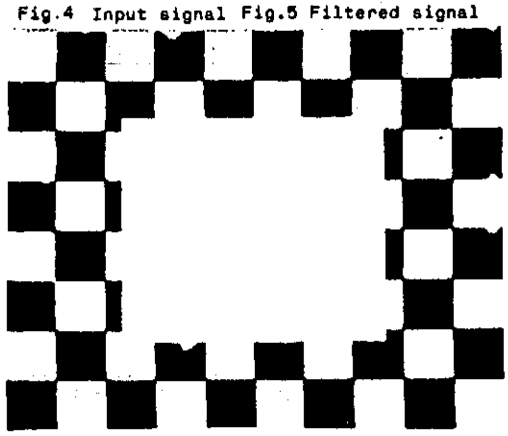
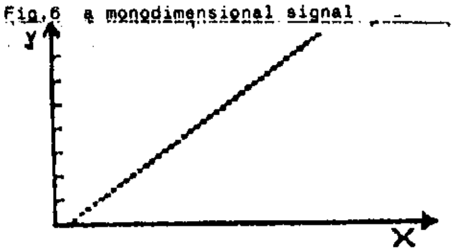


Fig.6 a monodimensional signal



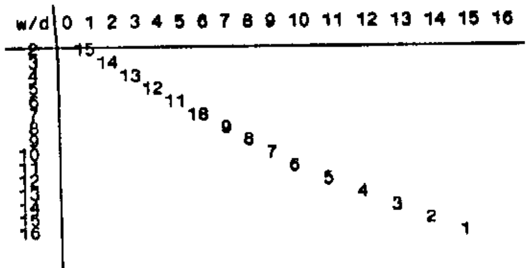


Fig.7 its matri of features

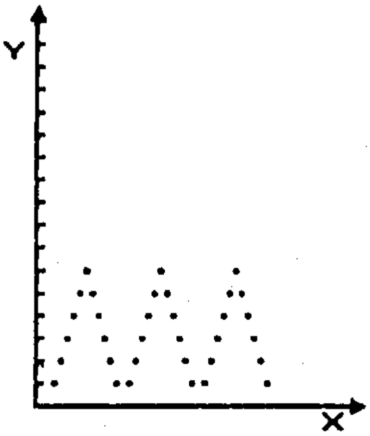


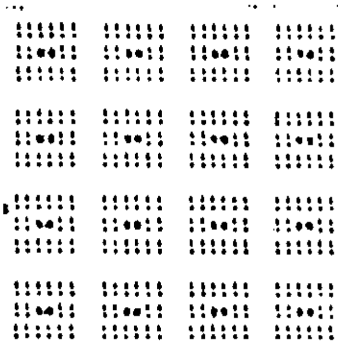
Fig.8 a monodimensional signal

w/d	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2			35														
3			5		29												
4					10		23										
5					5		10		17								
6							10		10		11						
7							5		10		10		5				
8									10		10		9				
9									5		10		13				
10											10		17				
11											5		21				
12													25				
13													24				
14													23				
15													22				
16													21				

Fig.9 its matrix of features

Fig.10

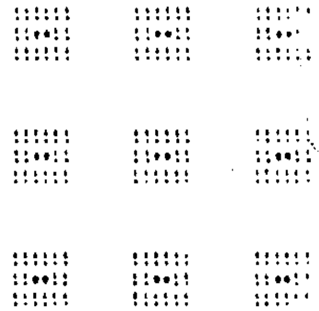
a two dimensional signal and its matrix of features



w/d	0	1	2	3	4	5	6	7	8	9	10	11
2						769						
3						325					351	
4											625	
5											570	
6											529	
7											484	
8											441	
9											482	
10											301	

Fig.11

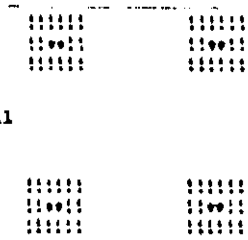
a two dimensinal signal and its matrix



w/d	0	1	2	3	4	5	6	7	8	9	10	11
2	483					361						
3	255					350					171	
4						405					324	
5						192					484	
6											625	
7											576	
8											529	
9											484	
10											441	

Fig.12

a two dimensional signal and its matrix



w/d	0	1	2	3	4	5	6	7	8	9	10	11
2	736					225						
3	578					219					105	
4	400					245					195	
5	200					287					289	
6						329					400	
7						147					529	
8											625	
9											576	
10											529	